Mathematics in this Lesson
Lesson 9

Lesson Description

Sasha and Keoni develop the vertex form of the equation of a parabola as $y = \frac{(x-h)^2}{4p} + k$ where the (h,k) is the vertex and p the distance from the vertex to the focus.

Targeted Understandings

This lesson can help students:

- Derive the vertex form of the equation of a parabola by using the method developed in previous lessons (which involves the definition of a parabola and the Pythagorean theorem), but generalizing from working with particular vertices to an unknown vertex (h,k) and generalizing from a specific distance from the vertex to the focus to an unknown p-value.
- Interpret algebraic expressions involving parameters ($x - h$, $y - k + p$, and $y - k - p$) as distances on a coordinate grid.
- Conceive of algebraic expressions, such as $x - h$, as both a single entity and as a process of subtracting the value of one variable from the value of another.

Common Core Math Standards

CCSS.M.HSG.GPE.A.2: Derive the equation of a parabola given a focus and directrix.

In this lesson, Sasha and Keoni build upon the method that they developed in Lessons 3 and 4 of using the geometric definition of a parabola and the Pythagorean theorem to derive the equations for particular parabolas with vertex at (0,0). They generalized this method in Lesson 5 for a family of parabolas with vertex at the origin but an unknown distance between the vertex and focus (the *p*-value). In Lesson 8, they generalized the method to parabolas with a specific non-origin vertex. Finally, in this lesson, they derive the vertex form for a parabola by using parameters (h,k) for the vertex and an unknown p-value.

CCSS.M.HSA.CED.A.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

The students build equations in two variables for different parabolas that capture a relationship between quantities. They represent these relationships on a coordinate grid system.

CCSS.M.HSA.SSE.A.1.B: Interpret complicated expressions by viewing one or more of their parts as a single entity.

On a parabola with general vertex (h, k) and unknown p-value, Sasha and Keoni express the distance from a general point (x, y) to its directrix as $y - k + p$. They conceive of this distance as a single entity, which they locate on the graph. They are also able to describe and locate the distances represented by parts of the expression: y, k, p, and $y - k$.

Common Core Math Practices

CCSS.Math.Practice.MP2: Reason abstractly and quantitatively.

According to the Common Core's description of Math Practice 2, mathematically proficient students are able to "*decontextualize*—to abstract a given situation and …manipulate the representing symbols as if they have a life of their own" and to "*contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved." In Episode 7, Sasha and Keoni use the Pythagorean theorem to set up the equation for a given parabola as $((y–k)–p)^2 + (x–h)^2 = ((y–k)–p)^2$ and then reason abstractly by performing appropriate algebraic transformations to arrive at the equation $y = \frac{(x-h)^2}{4p} + k$. However, they also reason quantitatively in Episode 6 by describing each term that they substituted into the Pythagorean theorem (namely, $x - h$, $y - k + p$, and $y - k - p$) as distances on the graph.