Mathematics in this Lesson
Lesson 6

Lesson Description

Keoni and Sasha compare the graphs of $y = \frac{x^2}{4p}$ for p-values of $\frac{1}{4}$, $\frac{1}{2}$, and 1. They figure out the effect that changing the value of p has on the graph of the parabola.

Targeted Understandings:

This lesson can help students:

- Connect geometry with algebra by verifying that a point belongs on a parabola in two ways: (a) using the geometric definition of a parabola, and (b) using algebraic substitution in the equation representing the parabola.
- Formulate the following relationships across comparable points for a set of parabolas, all with vertex at the origin, but with *p*-values of $\frac{1}{4}$, $\frac{1}{2}$, and 1:
  - When the *y*-value is fixed ($y = 4$), then the *x*-value increases as *p* increases.
  - When the *x*-value is fixed ($x = 2$), then the *y*-value decreases as *p* increases.
- Understand the following features of "special points" —points that align horizontally with the focus of a parabola:
  - The *y*-value of a special point is one-half its *x*-value.
  - The *x*-value of a special point is double its *y*-value.
  - The *y*-value of a special point is equal to the *p*-value of the parabola.
  - For a parabola with an unknown *p*-value, a special point can be expressed as $(2p, p)$.

Common Core Math Standards

CCSS.M.HSF.IF.C.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases

In an elaboration of this standard, the CCSSM learning trajectory for functions (Grade 8 and High School) states, "functions are often studied and understood as families, and students should spend time studying functions within a family, varying parameters to develop an understanding of how the parameters affect the graph of a function and its key features." In this lesson, Keoni and Sasha graph (by hand) a set of parabolas with increasing p-values of $\frac{1}{4}$, $\frac{1}{2}$, and 1. They figure out that increasing the *p*-value in the family of parabolas with vertex at the origin (and as represented by the function $y = \frac{x^2}{4p}$) results in the parabola getting wider on the same coordinate grid.

Common Core Math Practices

CCSS.Math.Practice.MP6: Attend to precision.

According to the Common Core's description of Math Practice 6, "mathematically proficient students try to communicate precisely to others." In this lesson, Sasha and Keoni improve in their ability to speak with precise mathematical language. For example, in Episode 5, they begin by stating the following inaccurate relationship: "When you change the value of *p*, the parabola gets wider" [**0:38]**. They then refine the statement to one that is more accurate: "As *p* increases, the parabola widens" [**0:47**]. In a second example, also from Episode 5, Sasha and Keoni are asked to articulate what they notice about the three "special points" that they have identified: $(\frac{1}{2}, \frac{1}{4})$, $(1, \frac{1}{2})$, and (2,1). At first, Keoni and Sasha report that "the *x*-value is, it's half of it, right, it's 2 and then 1" [**5:37**]. However, as they continue to work, they are able to more precisely convey that they are halving the *x*-value to obtain the *y*-value [**6:15**], which is not the same as saying that the *x*-value is one-half the y-value. They also re-express this relationship as the *x*-value of a special point being double its *y*-value [**7:07**]. This precision of language contributes to Sasha and Keoni's ability to eventually express the special point for a parabola with unknown *p*-value as (2*p*, *p*) [**8:02 – 8:20**].