Mathematics in this Lesson
Lesson 5

Lesson Description
Keoni and Sasha determine a general method for representing the equation for a parabola with a vertex on the origin and a focus *p* units above the vertex.

Targeted Understandings:

This lesson can help students:

- Interpret the meaning and use of an equation that they derive to represent a family of parabolas with vertex at the origin and focus *p* units above the vertex (y = $\frac{x^2}{4p}$).
- Conceive of the parameter *p* in the equation y = $\frac{x^2}{4p}$ as a distance between the vertex and focus of a parabola. The value of *p* can vary, but once it is set (e.g., when p = 2), then an equation is defined (e.g., y = $\frac{x^2}{8}$) that represents a unique parabola.
- Relate geometric features of parabolas to elements of corresponding equations. For example, the equation $y = \frac{x^2}{8}$ can also be expressed as $y = \frac{x^2}{2 \cdot 4}$, where the 2 represents the number of units between the focus and vertex of the parabola.

Common Core Math Standards

- **CCSS.M.HSG.GPE.A.2: Derive the equation of a parabola given a focus and directrix.**

  In Episode 6, Sasha and Keoni derive the equation (in two forms) for the family of parabolas with vertex (0,0) and focus *p* units above the vertex, namely y = $\frac{x^2}{4p}$ and x = $\sqrt{4py}$. They do this by generalizing the method they used to develop an equation for a parabola with *p* = 1 in Lessons 3 and 4 (y = $\frac{x^2}{4}$ and $x = \sqrt{4y}$), for a parabola with *p* = 2 in Episode 2 of this lesson (y = $\frac{x^2}{8}$ and $x = \sqrt{8y}$), and for a parabola with *p* = 3 in Episode 4 (y = $\frac{x^2}{12}$ and $x = \sqrt{12y}$).

- **CCSS.M.HSA.APR.A.1: Perform arithmetic operations on polynomials.**

Sasha and Keoni multiply binomials, such as $(y + 2)^2$ and $(y + p)^2$, in service of deriving an equation for a particular parabola (in Episodes 2 and 4) or a family of parabolas (Episode 6).

Common Core Math Practices

CCSS.Math.Practice.MP7: Look for and make use of structure.

In this lesson, Sasha and Keoni make use of mathematical structure on two levels. First, they discern a pattern relating the equations of three parabolas (y = $\frac{x^2}{4}$, y = $\frac{x^2}{8}$, and y = $\frac{x^2}{12}$) with the corresponding distance between the vertex and focus (respectively, p = 1; p = 2; and p = 3). Specifically, they see the structure of 1•4, 2•4 and 3•4 in the denominators of their equations, which is the *p*-value of the parabola multiplied by 4. Consequently Keoni and Sasha conjecture that the general equation of a parabola with vertex (0,0) and focus *p* units above the vertex will be y = $\frac{x^2}{4p}$. Then they look for and make use of structure on a second level. To derive the equation y = $\frac{x^2}{4p}$, they need to identify the lengths of the sides of a right triangle with hypotenuse connecting a general point on the parabola with the focus. To accomplish this challenging task, Sasha and Keoni see a pattern in similar quantities that they identified for particular parabolas, which they generalize to define lengths involving the parameter *p*, such as $y – p$ and $y + p$.