Mathematics in this Lesson
Lesson 3

Lesson Description

Developing an Equation for a Parabola Given Any *y*–Value
Keoni and Sasha create a general method for representing the x-value for any point on a particular parabola, given the y-value of that point. By using their previous results, along with the Pythagorean theorem, they are able to determine the equation for the parabola.

Targeted Understandings:

This lesson can help students:

- Interpret the meaning and use of an equation that they derive to relate the x-value to the y-value for a general point on a given parabola ($x = \sqrt{4y}$).

- Conceive of algebraic symbols, such as *x* and *y*, as quantities that have infinitely many possible values and that vary together, rather than only as unknowns that have a single numerical value.

- Express important quantities, such as the distance between a general point on a given parabola and its directrix, by generalizing one's reasoning from Lesson 2, when students determined the x-value for different points on the parabola when y was 5, 7 and 10.

Common Core Math Standards

- **CCSS.M.HSG.GPE.A.2: Derive the equation of a parabola given a focus and directrix.**

  Sasha and Keoni use the definition of a parabola, along with the Pythagorean theorem, to derive the equation of a given parabola in Lesson 3. They express it in a form that is useful for locating the x-value for any point on the parabola given its y-value (namely, $x = \sqrt{4y}$). In Lesson 4, they re-express and derive the equation as $y = \frac{x^2}{4}$. Later, they generalize their reasoning to derive the equation for any parabola with vertex (0,0) in Lesson 5 and any parabola with vertex (h,k) in Lesson 9.

- **CCSS.M.HSA.SSE.A.1.B: Interpret complicated expressions by viewing one or more of their parts as a single entity.**

Sasha and Keoni express the distance from a general point (x,y) on a particular parabola to its directrix ($y = -1$) as $y + 1$. They conceive of this distance as a single entity, which they locate on the graph. They are also able to describe and locate the distances represented by parts of the expression: y and 1. Similarly they interpret the expression $y - 1$ both as an entity and in terms of the distances represented by each of its parts.

Common Core Math Practices

CCSS.Math.Practice.MP2: Reason abstractly and quantitatively.

According to the Common Core's description of Math Practice 2, mathematically proficient students are able to "*decontextualize*—to abstract a given situation and …manipulate the representing symbols as if they have a life of their own" and to "*contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved." Sasha and Keoni use the Pythagorean theorem to set up the equation for a given parabola as $(y-1)^2 + x^2 = (y-1)^2$ and then reason abstractly by performing appropriate algebraic transformations to arrive at the equation $x = \sqrt{4y}$. However, they also reason quantitatively by describing the distances represented by each term in both equations: $y-1$, x, $y+1$, y, and $\sqrt{4y}$. They also discuss what the equation $x = \sqrt{4y}$ means and how it is useful.