Lesson 6, Proportional Reasoning Unit
Instructor Notes

Preparation:

- Print copies of the activity sheet to distribute to groups.
- It will also be helpful to have a document camera for groups to use to share their work on the activity.

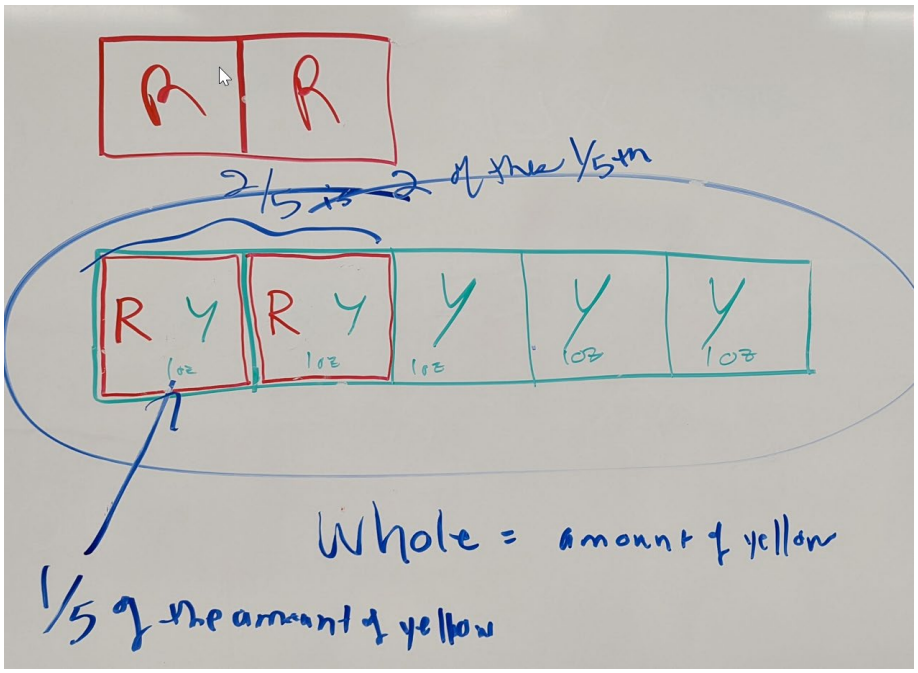
Lesson:

1. Introduction to Lesson 6

- Follow Slides 2-7 to introduce the lesson

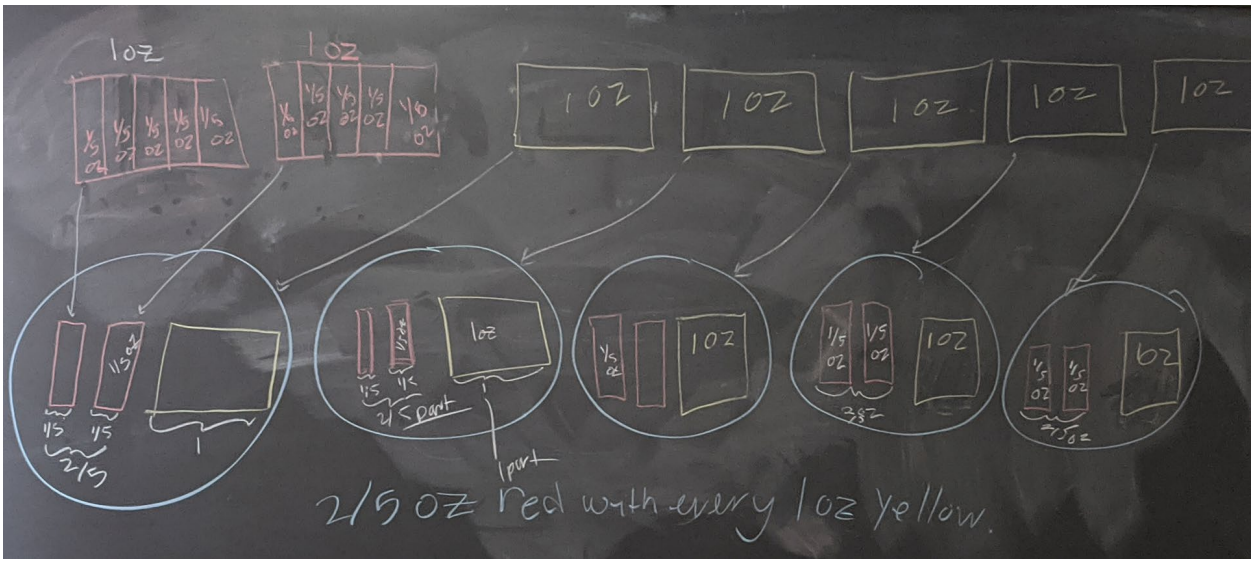
2. Activity: Interpreting Ratios as Fractions

- Distribute the activity sheets
- Present the task using Slide 9
- Circulate as groups work. This can be a challenging task.
    - You may need to remind groups of the following:
        - Make sure you find **two-fifths** not **five halves**.
        - The **two-fifths** should show in your drawings
    - A central part of identifying a fraction is identifying the whole. Thus, when students say they have found 2/5, ask them to show you the whole. 2/5 is two-fifths of some whole.
    - Preservice teachers typically find the interpretation that uses composed unit reasoning the hardest (see Sample Response below for Interpretation #2). They need to join the 2 oz red paint and 5 oz yellow paint together (to form a composed unit) and then split that new unit into 5 equal parts. Figuring out how to split 2 oz into 5 equal parts with a drawing can be challenging, and they also need to split the 5 oz into 5 equal parts.
    - **Sample Response, Interpretation #1**: The amount of red paint is always **2/5** (two-fifths) the total of the amount of yellow paint



First we showed the ratio of 2 ounces of red paint for every 5 ounces of yellow paint. Each square is one ounce. So on top, the 2 squares show 2 ounces red paint. On the bottom the 5 squares show 5 ounces yellow paint. Then we thought of the amount of yellow paint as the whole. Because the amount of yellow paint is already split into 5 equal parts, each part (i.e., each ounce) is 1/5 of the amount of yellow paint. So 2 ounces is 2/5 of the amount of yellow paint. But 2 ounces is also the amount of red paint (which we showed by labeling 2 of the squares with both R and Y). So, the amount of red paint is 2/5 the amount of yellow paint.

- **Sample Response, Interpretation #2**: You need **2/5** oz (i.e., two-fifths of an ounce) of red paint for every 1 ounce of yellow paint.



First we drew 2 ounces of red paint (the 2 red rectangles on top) and 5 ounces of yellow paint (yellow rectangles on top). We think of that as a batch (or composed unit). Then we want to split the entire batch into 5 equal pieces. Each of the 5 pieces is represented by a blue circle below. Splitting the 5 oz of yellow into 5 equal pieces is easy – each piece (a blue circle) gets 1 oz of yellow paint. But splitting the 2 oz of red paint into 5 equal pieces is harder. We split each ounce into 5 equal parts of 1/5 oz each and gave each blue circle a 1/5 oz. Then we repeated this for the second ounce of red paint. In all, each blue circle has 2/5 oz of red paint for every 1 oz yellow paint.

- Select presenters
- While presenters share, you can ask the rest of the class:
  - Is this an interpretation of the ratio 2/5 as a fraction two-fifths? Why or why not?
  - What's the whole?
  - What do you think is really effective about the drawing and/or explanation? What could be strengthened?

3. Conceptions of Fractions

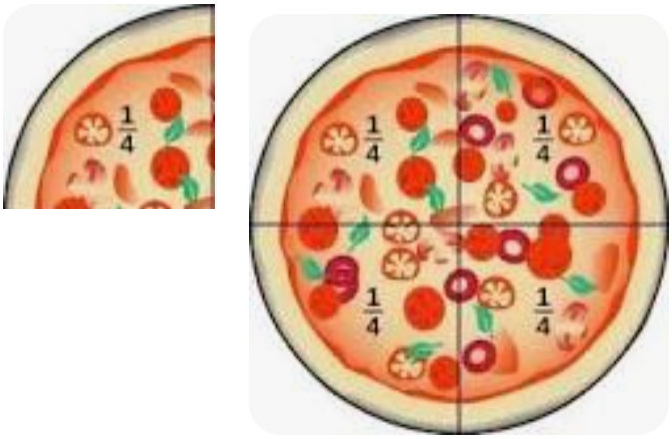
- Follow Slides 12 – 15 of the Class PowerPoint. This section is helpful for students when they work on reinterpreting a ratio as a fraction. It helps problematize the common instruction of saying that a fraction, like ¾ is "3 out of 4 parts," since that "out of' conception doesn't generalize well to improper fractions, like 5/4.

- **Sample Responses for Slide 14**

  - The first student might be thinking that you can't shade more parts than the total number of parts that you have
  - The second student seems to have turned the task into drawing 4/5 rather than 5/4, maybe because it's the only way you can shade a certain number of parts out of a total number of parts
  - Both students seem to be applying the idea that a fraction is a certain number of parts out of a total number of parts, which implies that you can't have a (improper) fraction greater than 1.

- **Sample Response for Slide 15**

  Each slice of pizza is ¼ of 1 pizza, and there are 5 slices.
  5/4 means 5 one-fourths of a pizza



- **Sample Response for Slide 16**

  - **Question 1:** What meaning for 5/4 did you use?

    Answer: 5/4 means 5 one-quarters or 5 one-fourths

  - **Question 2:** How does that differ from "5 out of 4"?

    Answer: You can have as many one-fourths of a pizza as you have pizzas, but 5 out of 4 suggests you can't have more pieces than you have parts.

4. **Optional Example of Interpreting a Ratio as a Fraction**

Follow Slides 18-22 of the Class PowerPoint for a second example of interpreting a ratio as a fraction. At the end of Slide 18, you can give your students a chance to come up with two different interpretations on their own. Then present an illustrated summary of the two interpretations using Slides 19-22.

5. Homework 6

Follow Slide 23 of the Class PowerPoint