

EXPLANATION A

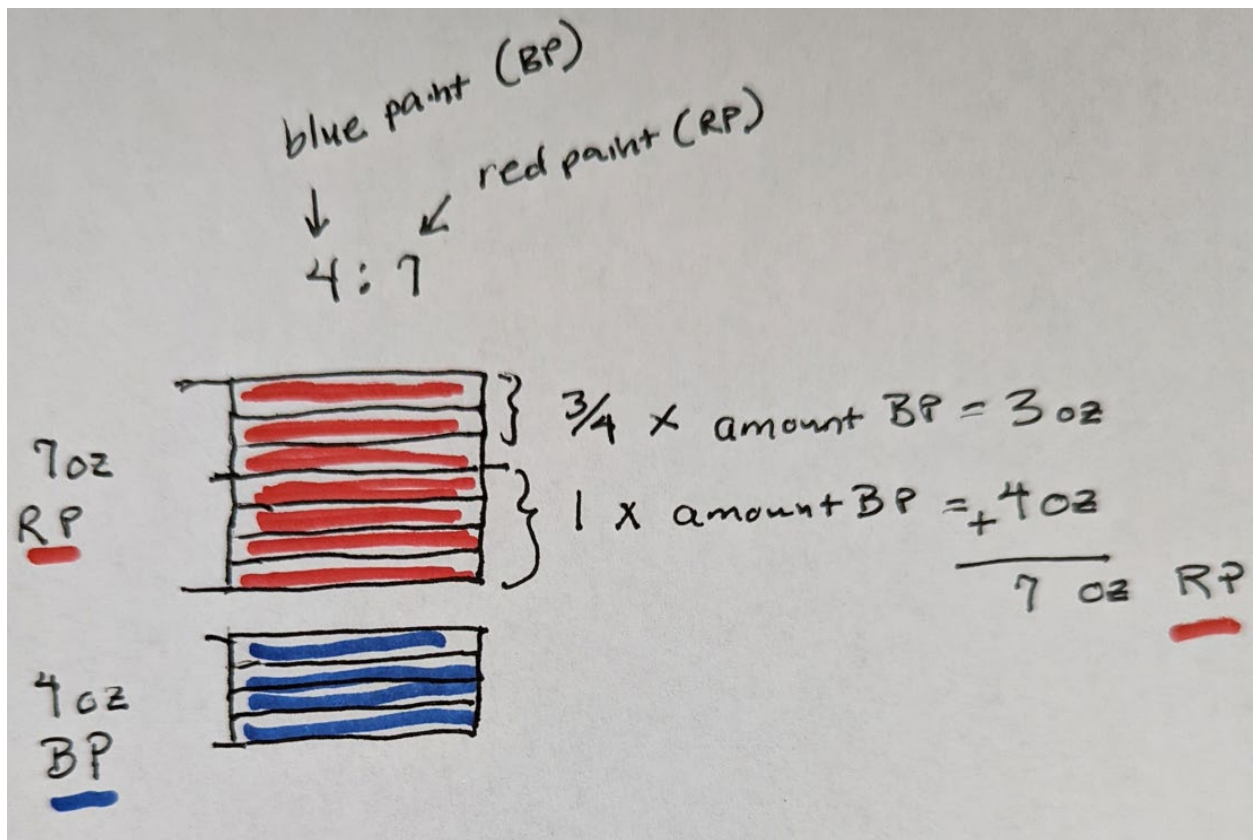
Task Given to Pre-service Math Teachers:

Suppose you create Batch 1 of purple paint using 4 ounces blue paint and 7 ounces red paint. You want to make a larger batch of paint (Batch 2) with the same purple shade. How much red paint should you use with 10 oz blue paint?

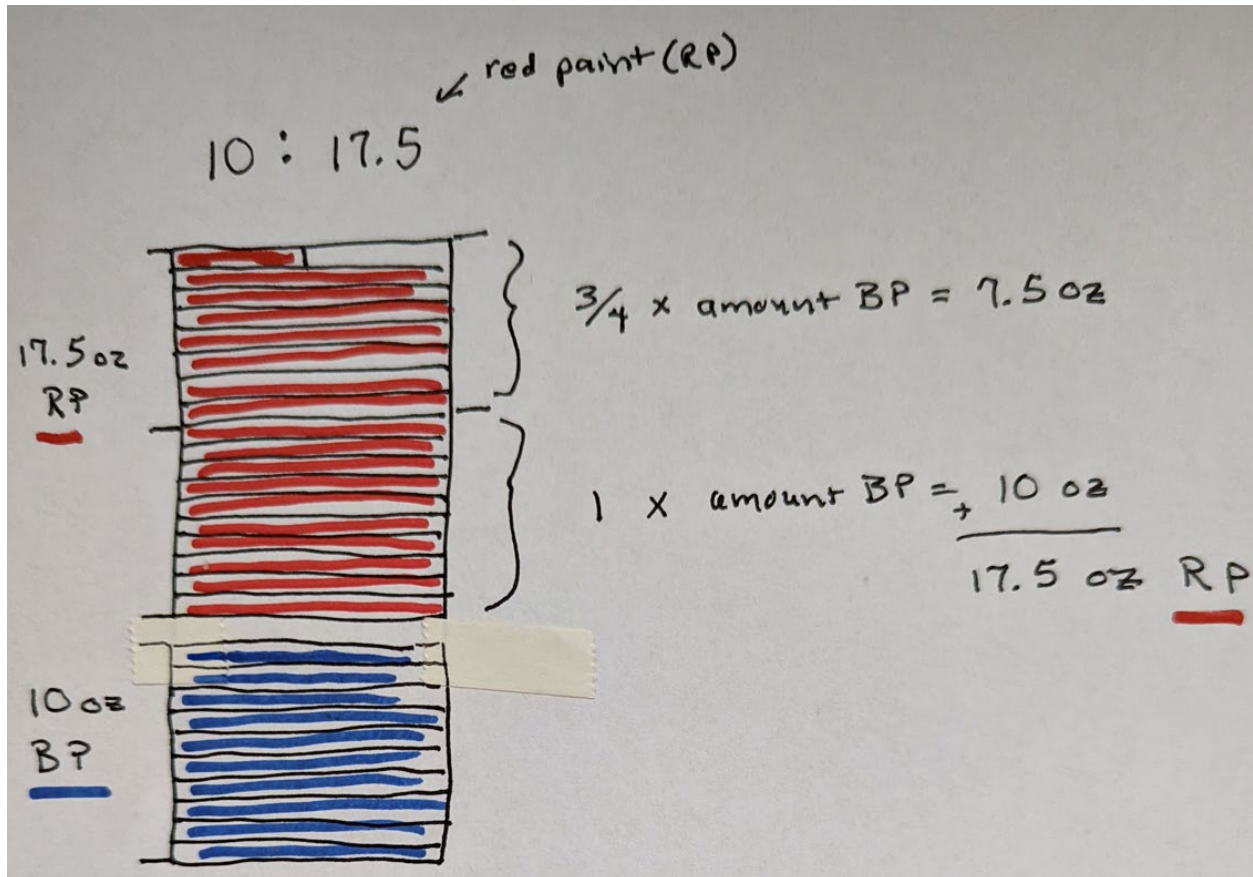
Solve this problem by first making a multiplicative comparison of the amount of red paint to the amount of blue paint in Batch 1. Use a drawing and explain your thinking in a way that you think would be effective with students. Don't perform any calculations separate from the drawing. Then apply your multiplicative comparison from Batch 1 to figure out how much red paint to use in Batch 2, by making a second drawing and explanation.

Response:

In the diagram below, we can see that there's $1\frac{3}{4}$ times as much red paint as blue paint:



I know that for Batch 2, there still must be $1\frac{3}{4}$ times the amount of red paint as blue paint. In the diagram below, we can see that there's $1\frac{3}{4}$ times as much red paint as blue paint. The reason for this is that 1 times the amount blue paint + $\frac{3}{4}$ times amount blue paint = 10 oz + 7.5 oz = 17.5 oz.



Thus, I'll need to mix 17.5 oz of red paint with the 10 oz of blue paint to get the same purple shade.

EXPLANATION B

Task Given to Pre-service Math Teachers:

Suppose a car travels 9 miles in 12 minutes at a constant speed. How far did the car travel in 8 minutes?

Solve this problem with **composed unit reasoning**. Create a **drawing** that you think would be appropriate to use with middle school students. **Explain** your thinking and the drawing in a way that you think would be effective with students. Don't perform calculations separate from the drawing; use the drawing to solve the problem.

Response:

The response is written on lined paper and consists of three parts:

1. A sequence of equations: $9:12 \rightarrow \frac{9}{12} = 1 \rightarrow \frac{3}{4} = 1 \rightarrow \frac{3}{4} \text{ mile for every } 1 \text{ minute}$

2. A number line diagram representing the rate. The line is divided into 8 equal segments by vertical tick marks. Above each segment is the label $\frac{3}{4} \text{ mile}$, and below each segment is the label 1 minute or 1 min .

3. A final calculation: $\frac{3}{4} \text{ mile/min} \cdot 8 \text{ min} = 6 \text{ miles}$

Explanation C

Task: Homework 4, Question 4

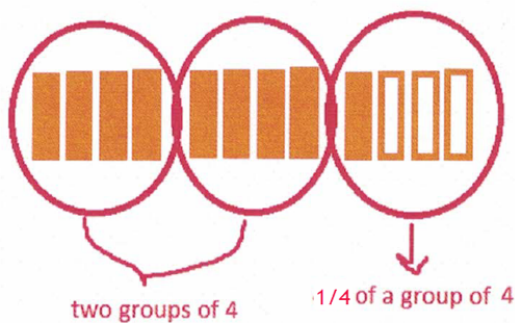
A zookeeper would like to make a larger batch of monkey chow that will taste the same as the original batch (of 4 cups of pumpkin and 9 cups of oatmeal). If she uses 10 cups of pumpkin, how much oatmeal should she use?

Solve this problem using a **multiplicative comparison** of oatmeal to pumpkin. Show in a **drawing** how many times more oatmeal there is than pumpkin in the original recipe. Then use this multiplicative comparison to make another drawing to solve the problem. Create well-labeled drawings. **Explain** your thinking.

Response: This is Explanation 1 from Lesson 5

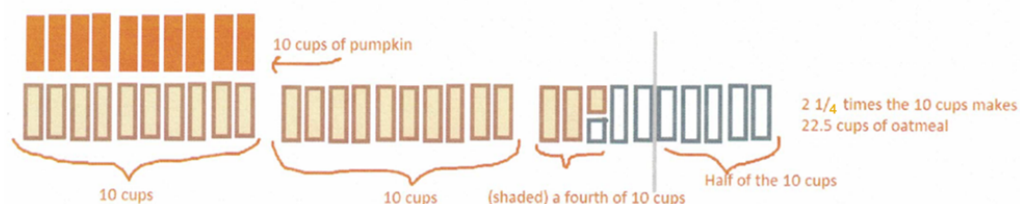
Let's start by thinking about the batch that has 4 cups of pumpkin and 9 cups of oatmeal.

We want to know how many times more oatmeal there is than pumpkin. It takes 2 groups of the 4 cups of pumpkin, plus $\frac{1}{4}$ of a group of 4 cups of pumpkin to equal the same amount of oatmeal. There are $2\frac{1}{4}$ times as much oatmeal as there is pumpkin.



Now we want to make a larger batch of monkey chow that follows the same recipe. If we use 10 cups of pumpkin, we will need $2\frac{1}{4}$ times as much oatmeal.

Two groups of 10 cups is 20 cups. To find $\frac{1}{4}$ of a group of 10 cups, we can split 10 cups in half, to get 5 cups. Then we split 5 cups in half to get $2\frac{1}{2}$ cups. Altogether we need 22.5 cups of oatmeal.



Explanation D

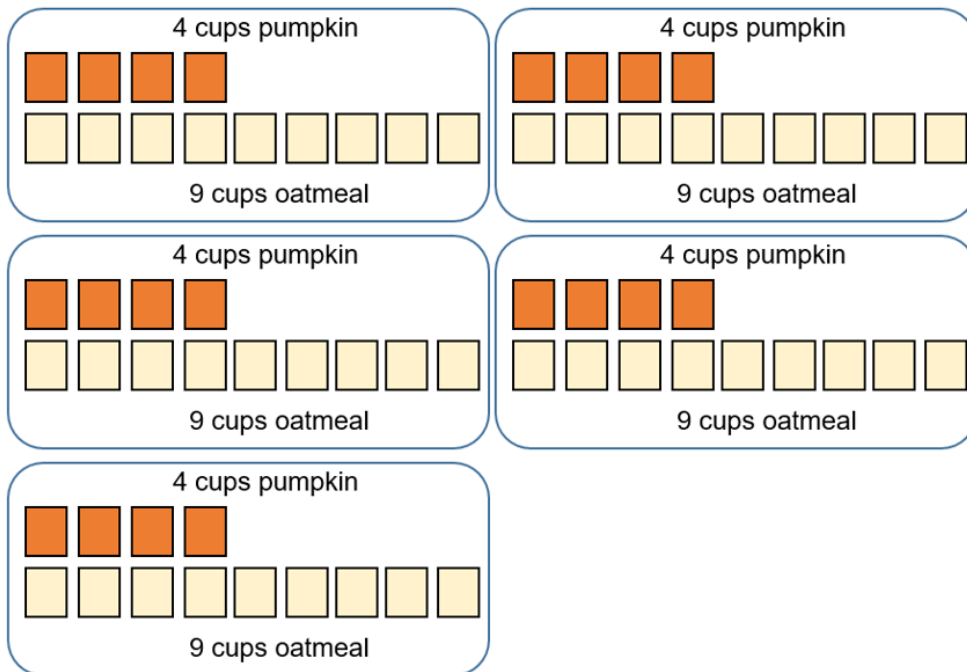
Task: Homework 3, Question 5

A zookeeper made a batch of Monkey Chow from 4 cups of pumpkin and 9 cups of oatmeal. The zookeeper would like to make a larger batch of monkey chow that will taste the same as the original batch. If she uses 21 cups of pumpkin, how much oatmeal should she use?

Solve this problem using **composed unit** reasoning. Create a **drawing** and **explain** your thinking in a way that you think would be effective with students.

Response:

We start with our recipe of 4 cups of pumpkin and 9 cups of oatmeal and just keep repeating making that batch until we have close to 21 cups of pumpkin. If we repeat the recipe 5 times, we will have 20 cups of pumpkin and 45 cups of oatmeal.



We only need $\frac{1}{4}$ more cup of pumpkin. If we split our recipe into four equal parts, one of those parts would have 1 cup of pumpkin. It would be so much easier if our recipe called for 8 cups of oatmeal. Then if we split the recipe into fourths, each part would have 2 cups of oatmeal and 1 cup of pumpkin. That leaves 1 cup of oatmeal. We will just split 1 cup into four equal parts of $\frac{1}{4}$ cup each and then give each of part of the recipe an extra $\frac{1}{4}$ cup oatmeal. That means we need $2\frac{1}{4}$ cups oatmeal for that extra cup of pumpkin. To answer the question, we add $45 + 2\frac{1}{4}$ to give us $47\frac{1}{4}$ cups oatmeal.

