Lesson 5, Proportional Reasoning Unit
Instructor Notes

Preparation:

- Print copies of Activities 1 and 2 to distribute to groups.
- Print color copies of Explanations 1 and 2
- It will also be helpful to have a document camera for groups to use to share their work for Activity 2

Lesson

1. Introduction to Lesson 5

- Follow Slide 2 to introduce the lesson
- This lesson can be a pivotal one for pre-service teachers. By reading and critiquing explanations written by other pre-service teachers, they often gain an appreciation for the need to provide clearly-labeled diagrams and well-elaborated explanations, and to connect the explanation with the diagram.

2. Activity 1: *What Makes a Good Explanation?*

- Remind everyone of the task that will be used for Activity 1 (the task from Homework 4, Question 4), by using Slide 4
- Then distribute Explanations 1 and 2 as well as the Activity 1 worksheet. Both Explanations were written by pre-service secondary math teachers (math majors) and both use an approach of multiplicative comparison reasoning. However, Explanation 1 is an example of a fairly good explanation while Explanation 2 is wanting.
- After groups have had an opportunity to read and discuss the explanations, record the results of their analysis using Slides 9-11.

- **Sample Responses to Explanation 1 (Slide 9)**

  List every part of the explanation and drawing that is effective:
  - "2 groups of" is the same as multiplication as 2
  - Showed how to find ¼ of 10, by first halving and then halving again
  - Used the knowledge from the first part in the second part
  - Can see the multiplicative comparison in the drawing
  - Liked the color coding – orange (pumpkin) vs beige (oatmeal)

What could be improved?

- The 2 ¼ groups of 4 cups of pumpkin could be better connected to the oatmeal
- The explanation at the bottom of the ¼ of a group seemed redundant

- **Sample Responses to Explanation 2 (Slide 10)**

  List every part of the explanation and drawing that is effective:
  - They labeled pumpkin and oatmeal and it's colored coded
  - It's concise and straightforward
  - You could count the number of boxes to get the answer

  Identify several problems with the explanation and drawing:
  - Focuses more on calculation rather than bringing meaning with the drawings
  - Drawings are not consistent and to scale. On top they used a square to represent a cup. Then down below, they used a different size and different orientation for each cup
  - There's no apparent 2.25 multiplicative comparison in the drawing. There's a drawing of what's given and then it's like all of sudden there's a drawing of the answer.
  - No explanation of what 2.25 is and there's no visual representation of it
  - At the top, arranging the amount of cups of each, 4 and 9, into arrays isn't really relevant to this problem

- **Sample Responses to Comparing Explanations 1 and 2 (Slide 11)**

  - Explanation 1 is as if you are talking to the student and go more in depth; Explanation 2 is more the calculations
  - Explanation 1 uses the diagram to show their thinking; they use the picture to calculate; picture and calculations go hand in hand. Explanation 2 they calculate and then illustrate what they found
  - Explanation 1 you see a clear pathway of their thought process and where the ratios come from. In Explanation 2, even computationally, you don't see where the 2.25 comes from.

- Wrap up the activity by following Slides 12 and 13.

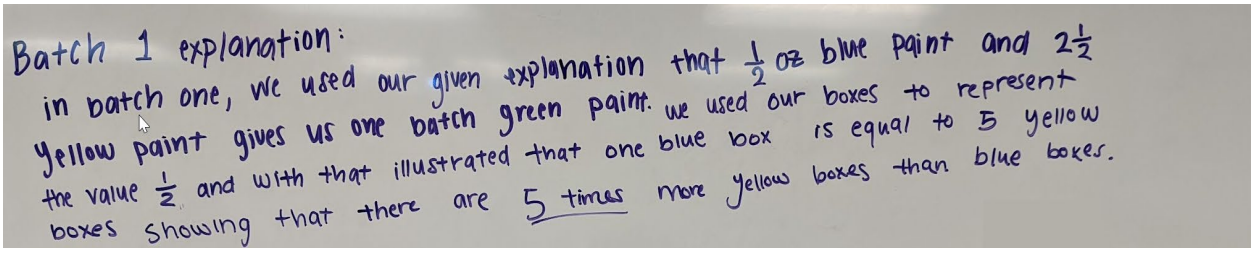
3. **Activity 2**

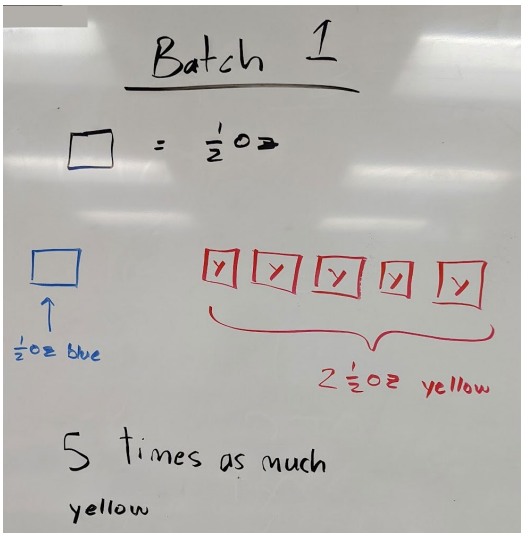
- Activity 2 provides an opportunity for pre-service teachers to apply what they learned about creating effective diagrams and explanations.
- One option is to distribute Activity 2A to half the groups and Activity 2B to the other half. Both activities utilize the same proportional reasoning task (Slide 16). However, groups given Activity 2A are asked to solve the task using multiplicative comparison reasoning and groups given Activity 2B are asked to solve the task using composed unit reasoning. This gives you an additional opportunity to later compare these two types of reasoning. Alternatively, you could use just Activity 2A or just 2B for all the groups and focus solely on creating effective drawings and explanations.
- Slides 16 – 18 can be used to present the task and activity
- Circulate as groups work. Select groups to share. Ask them to either put their drawing and written explanation on the board or use the document camera.
- After a group shares, ask other classmates to comment:

    - Ask if they need to hear the explanation again
    - What is effective about the explanation or drawing?
    - What would you change and way to make it even more effective?

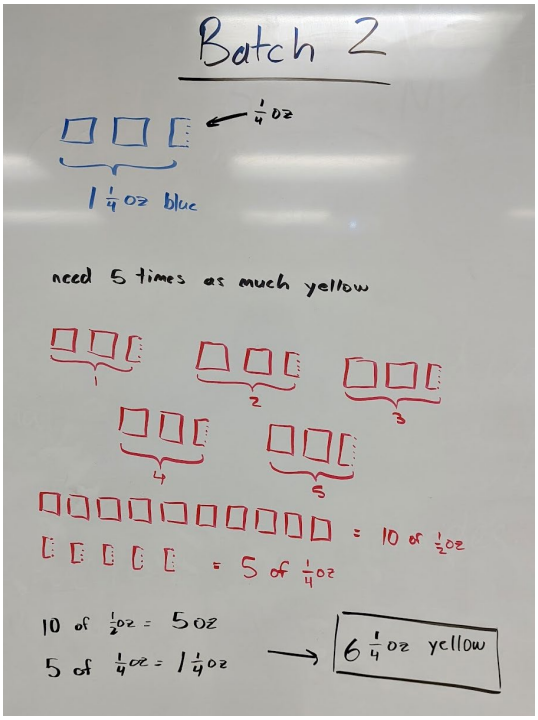
- **Sample Response to Activity 2A**

  Note that this explanation and drawing represents real progress from the class of pre-service teachers in which it was created, because the multiplicative comparisons can be seen in the drawings. However, there is still room for improvement, e.g., in the drawings, it would be hard for middle school students to see why a square represents ½ oz when there is no 1 oz drawing for comparison, and both explanations could benefit from additional elaboration.

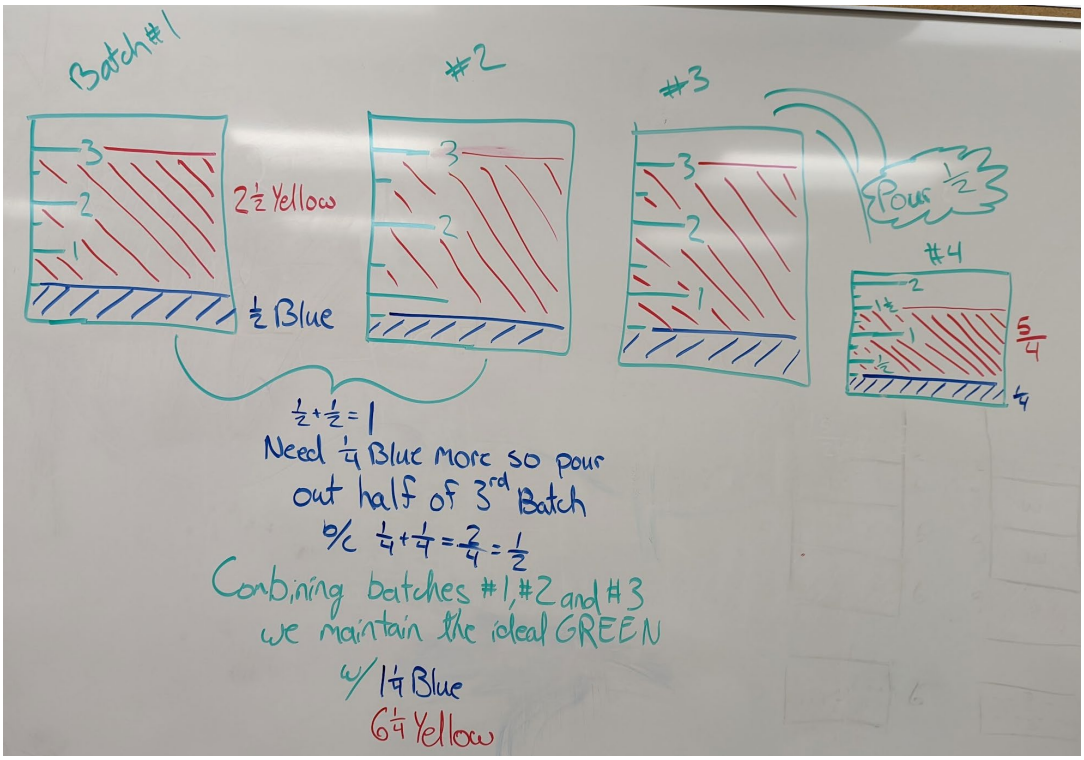

Batch 1 explanation:
in batch one, we used our given explanation that $\frac{1}{2}$ oz blue paint and $2\frac{1}{2}$ yellow paint gives us one batch green paint. we used our boxes to represent the value $\frac{1}{2}$ and with that illustrated that one blue box is equal to 5 yellow boxes showing that there are 5 times more yellow boxes than blue boxes.

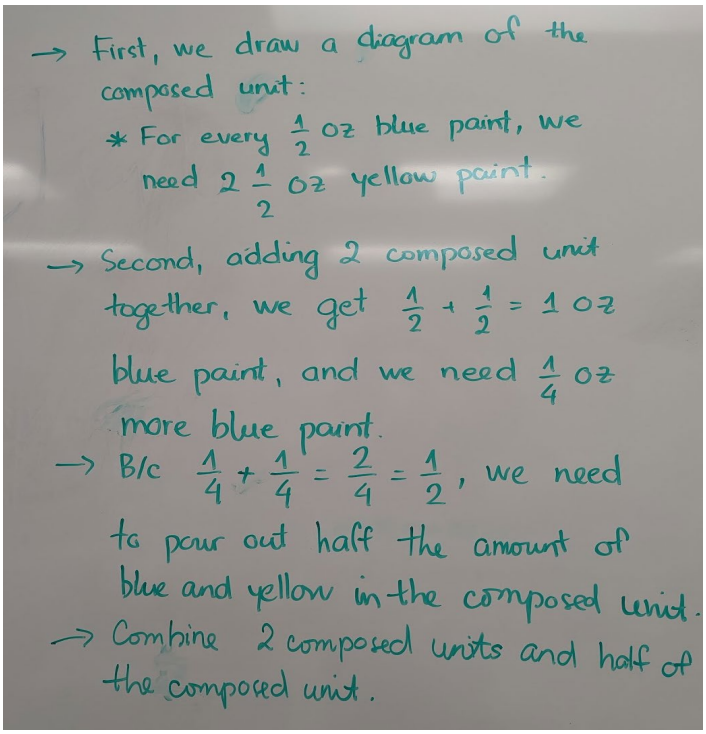


Batch 2:
Since we are given 1¼oz of blue paint, that translates to 2 and a half ½ oz boxes of blue paint.
And since we need 5 times as much yellow paint than blue paint, we need
5 groups of the same size.



- **Sample Response to Activity 2B**





- **Slide 20** can be used to compare differences between composed unit and multiplicative comparison reasoning. Sample Responses:

- For a multiplicative comparison, you are comparing one amount to the other amount (from the first batch); you're asking how many times greater is the amount of yellow paint than the amount of blue paint.
- For a multiplicative comparison you make a direct comparison of the two quantities (like yellow compared to blue paint). For composed unit reasoning you are operating on that unit – repeating, partitioning (splitting), and combining what you repeated with what you partitioned. All along you are preserving the relationship between blue and yellow paint even though you are not directly comparing blue to yellow paint.

4. **Homework 5**

- Follow Slide 21