Lesson 4, Proportional Reasoning Unit
Instructor Notes

Preparation:
- Print copies of Activities 1 and 2 to distribute to groups.
- It will also be helpful to have a document camera for groups to use to share their work for Activities 1 and 2

Lesson:

1. **Introduction to the Lesson**

   - Follow Slides 2 and 3

2. **Review Composed Unit Reasoning and Forming a Multiplicative Comparison**

   - **Slide 4** shows two tasks from Lesson 3. Ask class member to use these tasks and the work shown to explain in their own words what composed unit reasoning is
   - **Slide 5** is a review from Lesson 2. Sample responses:
     - Gianna has 2/3 the amount of candies that Alyssa has
     - Gianna has 2/3 as many candies as Alyssa
     - Alyssa has 1 ½ times the amount of candy as Gianna
   - Alyssa has 3/2 the amount of candy as Gianna

3. **View & Discuss Video**

   - Follow Slides 7 – 11 to set the context for the video that the class will watch. A few notes:
     - **Slide 7:** Ask someone to read the task aloud that Kate and Christopher were asked to solve
     - **Slides 8-9** introduce the Applet that Kate and Christopher use; You may want to go to https://www.geogebra.org/m/cnp4bbsy and explore the applet with the class
     - **Slide 10** summarizes how Kate and Christopher initially approached the task. The reason for using this slide rather than including an additional video, is to save time so that the class has time for Activity 2 later
   - Use **Slide 12** to show the video

- Use **Slide 13** to discuss the video. Sample responses:

  - They tried to make the drawing more proportional – their second drawing used dashed lines to show how they could fit equal amounts of 1.5 oz in the 4.5 oz
  - They showed 3 portions of the amount of red paint (1.5 oz) in the white paint

- **Slide 14** connects Kate and Christopher's second drawing to the multiplicative comparison that there are 3 times as much white paint as red paint

4. **Activity 1: Reasoning Proportionally with Multiplicative Comparisons**

- Goal: In this activity, your pre-service teachers apply the multiplicative comparison that Kate and Christopher formed (namely that the amount of white paint is three times the amount of red paint) to solve the original task (i.e., to find out how much white paints should be used with 2 oz of red paint in order to create paint that is the same shade of pink as the original batch)
- Distribute the Activity 1 worksheets
- You can use Slide 16 to present Activity 1. Ask someone to read the instructions aloud.
- Circulate as groups work. Watch for students who are working backwards (i.e., multiplying 2 x 3 in their heads and then drawing 6 oz) instead of thinking like a middle school student and using a drawing to solve.
- If a group finishes early, ask them to:
  - add a drawing of the original batch (Batch 1)
  - then describe what they'd like kids to notice about what is alike and what is different about the two batches [e.g., is there the same amount of paint in each batch or not? How is that reflected in your drawings?]
- To share, you may want to use a whiteboard or document camera
- One effective method for handling the sharing is to ask one group to share. Then ask other groups to say what they liked about the group's drawing and reasoning. Then see if any other groups have a different way of drawing that they'd like to share.
- A good question to pose to all the groups, "Can you see the multiplicative comparison of 3 in each drawing? How?"
- Sample drawings:
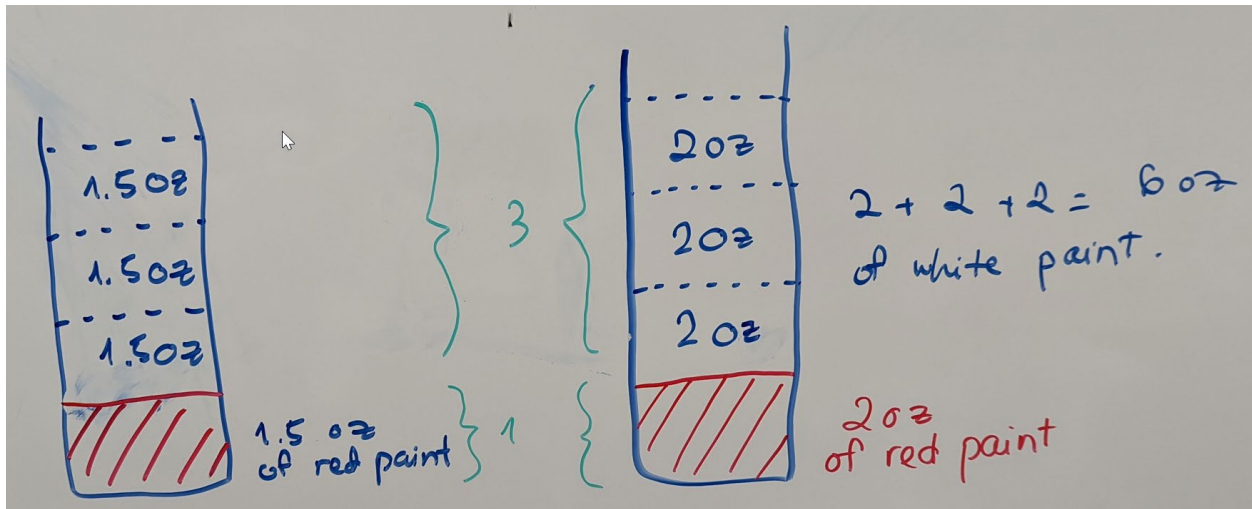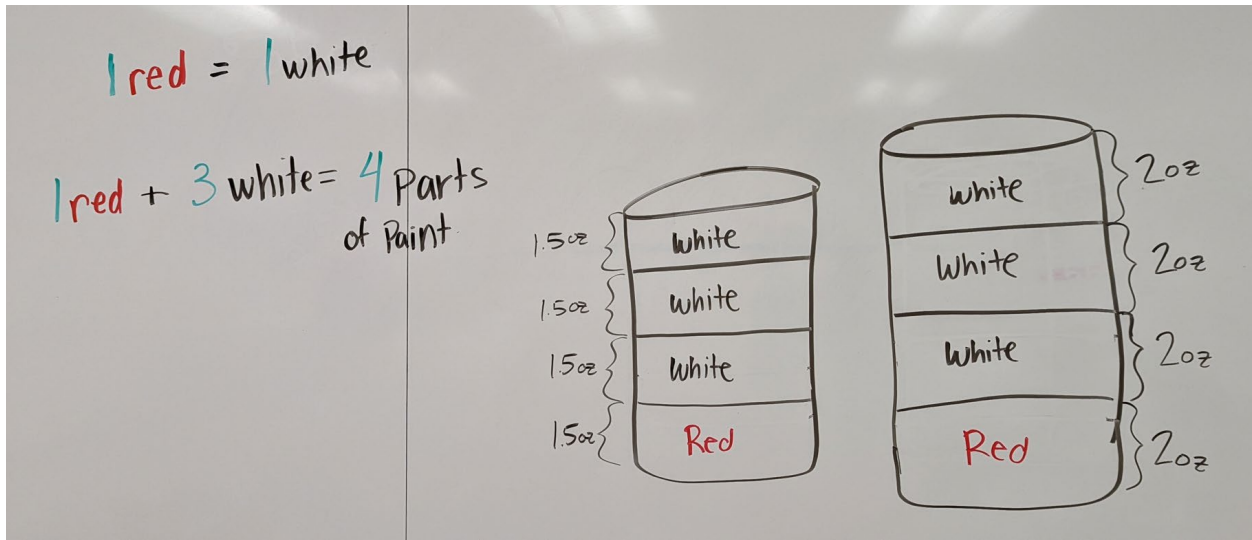
5. **Activity 2: Creating Drawings that Show Multiplicative Comparisons**

- Distribute the Activity 2 worksheets
- Display Slide 19. Ask someone to read the task at the top and the instructions through Question 1a.
- Discuss Question 1a together. The goal is to get to the response, "How many times more white paint is there than red paint" (or some equivalent, like *How many times greater is the amount of white paint than the amount of red paint? Or*

How many times greater is 14 oz than 6 oz?). This is really important. Without highlighting the question that one asks oneself when forming a multiplicative comparison, pre-service teachers tend to just start calculating and then have insufficient guidance for how to make their drawings.

- This task usually takes groups 20-25 minutes and can be challenging.

- Circulate as groups work:

  o You may need to remind them of the multiplicative comparison question: How many times greater is the amount of white paint (14 oz) than the amount of red paint (6 oz)?
  o Watch for groups who reduced fractions, e.g., 14:6 to 7:3 instead of forming a multiplicative comparison between 14 oz and 6 oz. You may need to ask them to repeat the question one asks when forming a multiplicative comparison: How many times greater is 14 oz than 6 oz? Then ask them how they can use a drawing to answer that question.
  o For groups that are stuck, encourage them to draw the amount of red paint (6 oz) and the amount of white paint (14 oz). Then ask, "What kind of comparison of these two amounts is this task asking for?"
  o When groups think they are done with Part 1a, ask them to SHOW you the multiplicative comparison of 2 1/3 in their drawing. It should pop out.
  o Some students get mixed up and form a composed unit instead of a multiplicative comparison
  o When moving to Question 2, watch for students who start calculating (e.g., 9 x 2 1/3) and encourage them instead to pretend they are middle school students and use a drawing to solve. You may also ask them what they are trying to show with their drawing (e.g., they need to draw an amount of paint that is 2 1/3 times as great as 9 oz of paint, or 2 1/3 groups of 9 oz)

- **Sample Response:**
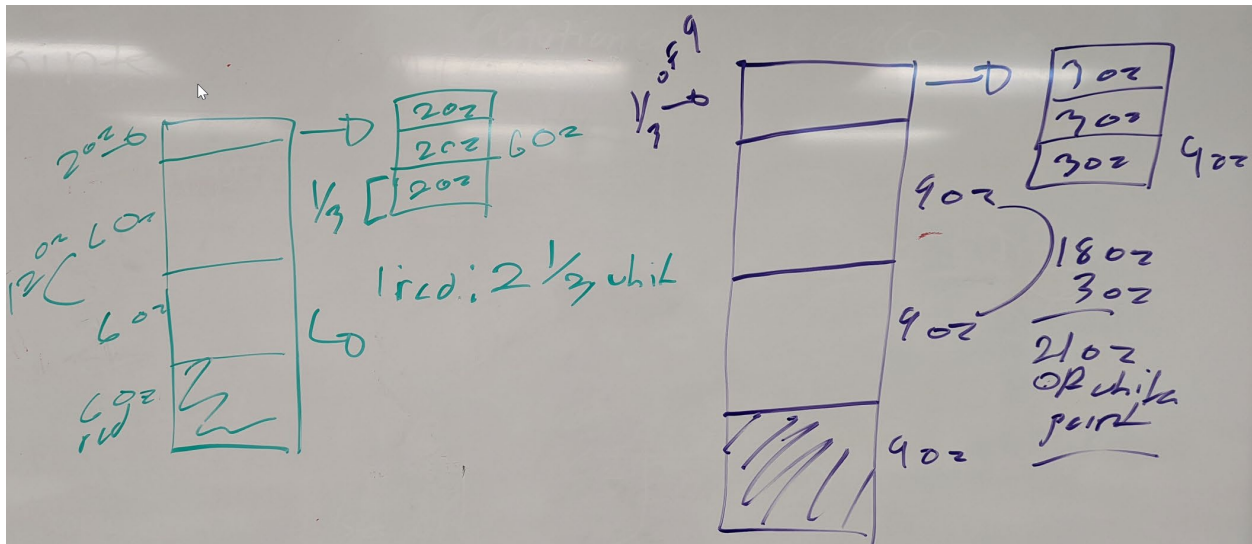
- The question was how many times greater is the amount of white paint than the amount of red paint. So we are trying to find how many times greater 14 oz is than 6 oz. Since we started with 6 oz, we broke the 14 oz into 2 groups of 6 oz, which is 12 oz. The remaining 2 oz is 1/3 of 6 ounces. We concluded that the amount of white paint is 2 1/3 times greater than the amount of red paint in Batch 1.
- So in Batch 2 the amount of white paint also needs to be 2 1/3 times greater than the amount of red paint, in order for the two batches to be the same shade of pink. We are given that there are 9 oz of red paint in Batch 2. So we need to figure out what 2 1/3 times 9 oz is. We drew 2 groups of 9 oz (which is 18 oz) and 1/3 of a group of 9 oz (which is 3 oz). The answer is that we need 21 oz of white paint (because 18 oz + 3 oz = 21 oz).

6. Homework 4

- Follow Slide 21