

Lesson 3, Proportional Reasoning Unit Instructor Notes

Preparation:

- Print copies of Activities 1-3 to distribute to groups.
- It will also be helpful to have a document camera for groups to use to share their work to Activity 3

Lesson:

1. Introduction to the Lesson

- Follow Slides 2-3

2. Video 1

- Use Slides 4-6 to set up the context for Video 1
- Show the 2-minute video using Slide 7

3. Activity 1: *Discussion of Activity 1*

- Distribute Activity 1 worksheets
- After groups have had time to discuss, ask groups to share their responses with the whole class
- **Sample responses to Question 1 (Slide 10)**
 - The amount of communication between the students
 - They seem like they know what they're doing; on the same page
 - When they performed calculations, they did it to both distance and time
 - They found that doubling keeps the speed the same
 - Kept the Lamborghini's distance and time the same in the simulation and the table
- **Sample responses to Question 2 (Slide 11)** – If students don't share their reasons but are not convinced that Kate and Christopher are reasoning proportionally, ask what they would like to see them do or say.
 - Yes, we think they are reasoning proportionally to an extent, because they figured out that doubling each value worked
 - We liked the fact that they figured out the 15 mile and 6 min, because it was not just doubling. But they didn't explain their reasoning

- Sort of convinced. Liked the doubling. Couldn't see where they got the 15 and 6 – wanted them to explain
- Wanted Kate to expand more on her reasoning. The only reason they came up with the doubling was that they were using the simulation.

4. **View and Discuss Videos 2 & 3**

- Use Slide 12 to show the surprising situation that occurred right after Video 1
- Ask your students for their conjectures about what Christopher could be thinking (Slide 12)
- Slide 13 lets your students know how long the video will be and what to watch for
- Play Video 2 using Slide 14 to present Christopher's strategy of dividing one number by 2 and multiplying the other number by 2.
- Use Slide 15 to ask your students to make conjectures why they think Christopher's idea is reasonable to him (especially in light of his previous, correct work)
 - Sample Responses:
 - Maybe Christopher is thinking that dividing by 2 and multiplying by 2 will balance themselves, like multiplying by 1 (or $2/2$)
 - Maybe Christopher is just throwing out a lot of numeric patterns and thinking they could all work, without considering whether or not the patterns are consistent with each other.
- View and discuss Video 3 (using Slides 16-18)
- Videos 2 and 3 help communicate to pre-service teachers that when all students are doing is noticing number patterns (without knowing why those patterns hold), some students will generate patterns that are inconsistent with each other. This is one way to problematize the importance of asking students to explore the underlying reason why some patterns hold (which Kate and Christopher attempt in Videos 4 and 5)

5. **Activity 2: Analyzing Kate's Drawing**

- Activity 2 is a discussion of Video 4, in which Kate creates a drawing in an attempt to explain **why** one of their number patterns works
- Use Slide 19 to motivate the task of asking Kate and Christopher to draw a picture that explains why traveling 20 miles in 8 minutes was the same speed as traveling 10 miles in 4 minutes.
- Use Slides 20-21 to set up and show Video 4
- Distribute Activity 2 worksheets

- After groups have had a few minutes to discuss, ask groups to share their responses with the whole class
- **Sample Responses to Question 1 (Slide 24)**
 - It illustrates the distance each person has to walk and a stopping point. It looks like both people have same speed for the first 10 miles because of the arrows
 - Shows a common check point (with the arrows) and it shows that the top person went farther
 - Kate’s drawing with her explanation showed how one person can stop and keep going and both be going the same speed
- **Sample Responses to Question 2 (Slide 25)**
 - the top distance doesn’t look double the 10 miles
 - the drawing needs some units and scaling
 - Kate needs to flesh out the part where the top guy keeps going
 - the drawing doesn’t show what has been doubled
 - need a bit more in the explanation – say they are going the same speed

6. Video 5 and Analyzing Kate’s Drawing 2

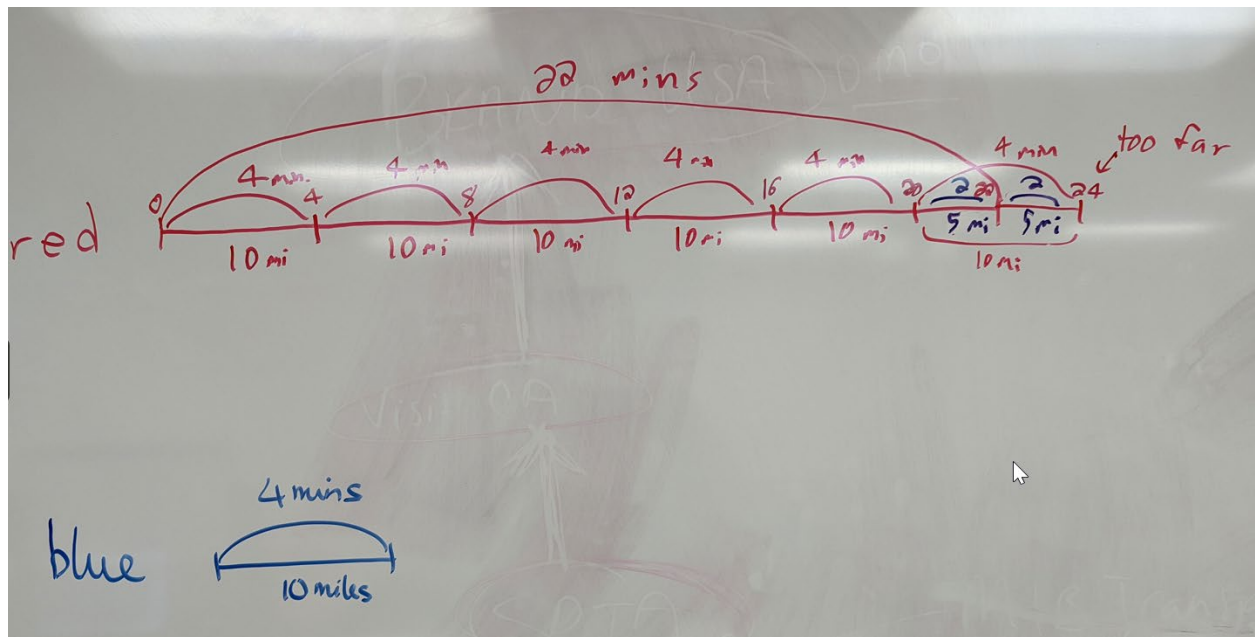
- Use Slides 26-27 to set up and show Video 5, in which Kate and Christopher are asked to make a new drawing
- Compare Drawing 1 and Drawing 2 using Slides 28-29
- **Sample responses to Slide 29:** What is different about Drawing 2?
 - They used numbers
 - It was more to scale
 - You could see that the journey by the guy on top was 3 of the little journeys of the guy on the bottom, and that the guy on the bottom’s journey was $\frac{1}{3}$ of the journey of the guy on top
 - You could see tripling in the diagram
 - The distance of 10 miles was combined with the time of 4 min into a chunk or “hop”
- Use Slide 30 to summarize the emergence of this new unit where Kate and Christopher seem to have joined the 10 miles with the 4 minutes; then they are able to repeat that unit. In the next slide, you will introduce terminology to name this new unit

7. Introduce the terminology of a COMPOSED UNIT

- Follow Slides 31-34 to define composed unit reasoning and present examples

8. Activity 3: Using Composed Unit Reasoning

- Distribute Activity 3 worksheets
- Display the task using Slide 36; ask a class member to read it aloud.
- Circulate as groups work. Make sure they NOT using the proportion algorithm first, calculating, and then illustrating the answer. They need to think like a middle schooler and use the drawing to arrive at the answer to the task.
- Groups can share using a whiteboard or document camera
- **Sample Response:**



The blue car traveled 10 miles in 4 minutes. The red car kept traveling in chunks of 10 miles in 4 minutes (5 chunks in all) until they had traveled 50 miles in 20 minutes. If we go one more chunk of 10 miles in 4 minutes, we will have gone 60 miles 24 minutes, which traveling too far and for too long. After traveling for 20 minutes, they need to go for 2 more minutes. So we split the chunk of 10 miles in 4 minutes into two equal parts of 5 miles in 2 minutes each. We added the 5 miles in 2 minutes onto the total of 50 miles in 20 minutes to conclude that the blue car went **55 miles** in 22 minutes

9. Homework 3

- Follow Slide 38