

## Lesson 2, Proportional Reasoning Unit Instructor Notes

### Preparation:

- Print copies of Activities 1-3 to distribute to groups.
- You may also want to double-check that the audio on your projection system is working; so that you can play the video on Slide 24 of the Class PowerPoint

### Lesson:

#### 1. Introduction to the Theme: Types of Non-Proportional Reasoning

- Follow Slides 2-9 of the Class PowerPoint

#### 2. Activity 1: Characterize a Type of Non-Proportional Reasoning

- Distribute Activity 1 worksheets to class members
- Display Slide 11 and ask someone to read Questions 1 and 2
- Point out to students that there is a reminder of the students' work on the Orange Juice Task and the Water Faucet Task on the back side of the Activity 1 worksheet
- Circulate as students work and select students to share with the whole class
- **Sample responses to Question 1** (Slide 12):
  - We noticed that the focus was on single values, e.g., just looking at glass size or just amount of time, rather than a relationship between amount of water and OJ concentrate or between amount of time and amount of water dripped. They are looking at just one component or factor instead of the whole.
  - Discrete explanation. Big glass – if just focused on OJ, conclude it will taste more orangey; if focused on water, could conclude the big glass will taste more watery
- **Introduce the Terminology UNIVARIATE REASONING**
  - Use Slide 13 to introduce and define that term univariate reasoning, as a type of non-proportional reasoning
  - Connect this terminology to your students' responses to Question 1 of Activity 1.

- **Sample responses to Question 2 of Activity 1 (Slide 14)**
  - Faster during certain times. Looking just at the water could conclude it's faster, slower, faster. 6 gal, 4 gal, 8 gal.
  - Could reason univariately just looking at time. 3 min, 2min, 4 min – slow, fast, slow
- Present a real 7<sup>th</sup> grader's response to the Pool Task, who exhibits univariate reasoning
  - Follow Slide 15
  - This data comes from the following research study:
 

Lobato, J., Hohensee, C., & Rhodehamel, B. (2013). Students' mathematical noticing. *Journal for Research in Mathematics Education*, 44(5), 809-850.

### 3. **Activity 2: What is Another Common Way that You Will See Students Reasoning Non-Proportionally? (Video)**

- Set up the context for the video that the class will react to in Activity 2 by using Slides 17-21
- Distribute Activity 2 worksheets before viewing the video, so that your students can see that they will have access to the transcript from the video.
- Slide 23 points out what they should watch for in the video
- Play the video from Slide 24
- Circulate as groups work
- Groups share. **Sample response to Question 1 (Slide 15):**
  - Kate added the same amount, 3 to each number.  $12 \text{ miles} + 3 = 15 \text{ miles}$ ;  $9 \text{ min} + 3 = 12 \text{ minutes}$
- **Sample responses to Question 2 (Slide 16)**
  - It feels like it's balancing things out, 3 and 3, like multiplying by 1.
  - Maybe it's comfortable; you learn addition and subtraction before multiplication and division

### 4. **Introduce Additive Reasoning and Additive Versus Multiplicative Comparisons**

- Before naming Kate’s strategy (as additive reasoning) solidify and generalize this non-proportional way of reasoning by exploring a second example (Slide 27)
- **Sample Responses to Question 1 from Slide 27:** What might Erika be thinking?
  - $10 - 7 = 3$ ; so  $4 - 3 = 1$
  - $10 - 4 = 6$ ; so  $7 - 6 = 1$
- **Sample Responses to Question 2 from Slide 27:** How is Erika’s reasoning similar to Kate’s? How is it different?
  - Both are not reasoning proportionally
  - They both are adding or subtracting the same amount to each number given
  - The main difference is that Kate used addition and Erika used subtraction
- **Introduce the Terminology ADDITIVE REASONING**
  - Use Slide 28 to introduce and define that term *additive reasoning*, as a second type of non-proportional reasoning
  - It is the reasoning that Kate and Erika both used
  - The definition of additive reasoning hinges on the notion of making two additive comparisons. An additive comparison is explored next (Slide 29) and contrasted to a multiplicative comparison (Slide 30)
- **Sample Responses to Slide 29:** Different ways to compare the number of rocks that two people have
  - Darius has 9 more rocks than Mauricio
  - Mauricio has 9 fewer rocks than Darius
  - Darius has 4 times as many rocks than Mauricio
  - Mauricio has  $\frac{1}{4}$  as many rocks as Darius

These are the four responses that you want to draw out.

You may also want to follow up asking if they can group the comparisons into ones that feel alike (e.g., the first two and the last two)

You may also have students who says that “Mauricio has 4 times less rocks than Darius” You may want to respond that it’s not very common in English to

hear a phrase like “4 times less” and it can result in confusion. Then ask if they can rephrase using a fraction.

- Use Slides 30 and 31 to define **additive comparison** and **multiplicative comparison**. It’s important to emphasize the types of questions that one asks oneself when mentally comparing amounts of things additively versus multiplicatively. Then you can later point out that it is NOT the case that “making an additive comparison” or “additive reasoning” is simply using addition and that “making a multiplicative comparison” is simply using multiplication. For example, the weights 7 lbs and 21 lbs can be compared additively by asking, “How much less is 7 lbs than 21 lbs” and using subtraction to answer ( $21 - 7 = 14$ ; 7 lbs is 14 lbs less than 21 lbs). Similarly, we can compare the two weights multiplicatively by using asking, How many times larger is 21 lbs than 7 lbs, and then using addition to figure it out ( $7 + 7 + 7 = 21$ ; so 21 lb is 3 times as large as 7 lbs)
- Slide 32 is a repeat of Slide 28. It’s helpful to revisit the definition of additive reasoning, once the terminology of additive comparison has been defined.
- Slide 33 is important because some of your students will like think, because of the name, that additive reasoning means the use of addition. But it’s not that simple. The reason for using this terminology, despite its potential for misperception, is because it is commonly used in the mathematics education literature.

## 5. Activity 3: Recognizing Additive Reasoning

- Distribute Activity 3 worksheets
- Ask someone to read the task aloud
- The goal of the task is to give your students an opportunity to think about how a student who is not reasoning proportionally, and specifically who is reasoning additively, might approach this task.
- Response to Activity 3 (Slide 36):
  - 7 is 1 minute less than 8 minutes, meaning you are subtracting 1 from 8. So we need to also subtract 1 from 6.  $6 \text{ ounces} - 1 = 5 \text{ ounces}$
  - $8 - 6 = 2$ ; so  $7 - 2 = 5$

## 6. Summary of Two Types of Non-Proportional Reasoning

- Use Slide 37

**7. Homework 2**

- Use Slide 38