

## Homework 2

**Read:** Pages 15 to the top of page 19 from the following book:

Lobato, J., & Ellis, A. B. (2010). *Essential understandings: Ratios, proportions, and proportional reasoning*. In R. M. Zbiek (Series Ed.), *Essential understandings*. Reston, VA: National Council of Teachers of Mathematics.

### 1. Univariate Reasoning

- What is univariate reasoning? Describe in your own words.
- Consider the following task: *Students are shown the ramp below and asked if the ramp is the same steepness throughout or whether it is steeper in places.*

How might a student who is reasoning univariately respond and why?



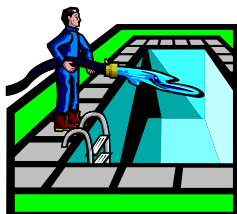
### 2. Additive versus Multiplicative Comparisons

- List two or more questions that people ask themselves when they are forming an *additive comparison* between two amounts.
- List two or more questions that people ask themselves when they are forming a *multiplicative comparison* between two amounts. Be sure to think about comparing both a larger amount to a smaller amount **and** a smaller amount to a larger amount.
- Lily ate 7 potato chips and Mel ate 21 chips. Write 2 statements, each that makes an additive comparison.
- Using the same context in Question 3c, write 2 statements, each that makes a multiplicative comparison.

**3. Additive Reasoning.** The term “additive reasoning” is used to refer to a way of thinking non-proportionally in a proportional situation. Students attend to both quantities in the situation (unlike univariate reasoning), but they make an absolute (additive) rather than relative (multiplicative) comparison between two amounts and then apply that same additive comparison to two other amounts in the situation.

- Consider the following task situation about a swimming pool

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Water is being pumped through a hose into a large swimming pool. The table shows the amount of water in the pool over time.

Time in minutes	Amount of Water in gallons
0	0
3	6
5	10
9	18

How might a student who is using *additive reasoning* respond to the following question: *How much water is being pumped in 1 minute?*

- b. Write a math task that is proportional in nature. Then present an imaginary student's response that shows additive reasoning.

*Notes:*

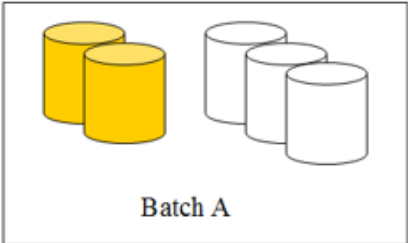
- *Don't just give the student's answer; provide their reasoning*
- *Their answer will be incorrect, because additive reasoning is non-proportional.*
- *Use a different task from what has appeared in class or the reading.*

### 4. Analyzing Student Reasoning: Cynthia

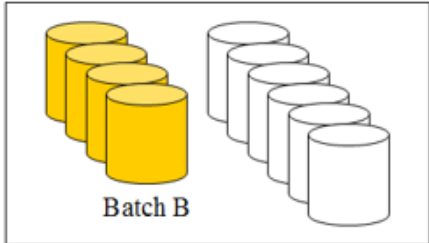
Below you will read what a Grade 7 student—Cynthia—said in response to the following math task, when she was interviewed by a researcher:

To make orange juice you mix cans of orange juice concentrate with cans of water.

You made Batch A of orange juice and your friend made Batch B. The shaded cans are filled with orange juice concentrate. The unshaded cans are filled with water.



Batch A



Batch B

**Which batch will taste “orangier” or will they taste equally “orangey”?**

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Identify which type of reasoning Cynthia is using: (a) univariate reasoning; (b) additive reasoning; (c) making a multiplicative comparison; or (d) proportional reasoning. **Explain** how you know.

Interviewer: [After they have talked about how to make orange juice] So do you think that these would taste equally orangey, or would one taste more orangey than the other?

Cynthia: Batch B because it has 4 gallons of orange juice, so it'll taste a little orangier because there's more orange juice. It'll taste orangier because it's more.

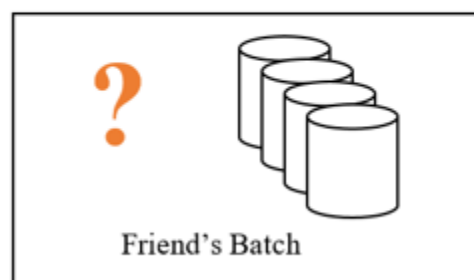
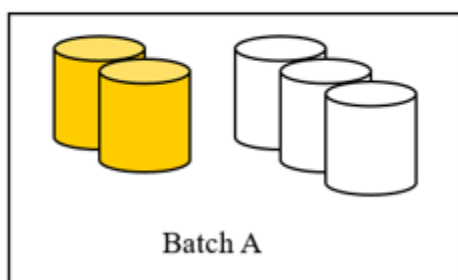
Interviewer: Okay, so let me just - I think I've got it right. Batch B will taste orangier than Batch A, and it's because you said there was more of the orange juice concentrate?

Cynthia: [nods "yes"].

### 5. Analyzing Student Reasoning: Diego

Below you will read what a Grade 7 student—Diego—said in response to the following math task, when he was interviewed by a researcher:

Suppose your friend wants to make a batch of orange juice that tastes the same as Batch A. How many cans of orange juice concentrate should your friend use with 4 cans of water?



Identify which type of reasoning Diego is using: (a) univariate reasoning; (b) additive reasoning; (c) making a multiplicative comparison; or (d) proportional reasoning. **Explain** how you know.

Interviewer: This time, we will start with the same Batch A that we used before. It is made with 2 cans of orange juice concentrate and 3 cans of water. Suppose that your friends want to make a batch of orange juice that

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tastes the same as Batch A. How many cans of orange juice concentrate should your friend use with 4 cans of water?

Diego: um (pause), 3

Interviewer: 3 cans of orange juice concentrate?

Diego: Yeah.

Interviewer: And what were you thinking?

Diego: This one (points to Batch A) has 1 less orange juice, like there's 3 (points to the 3 cans of water), so you minus 1, and there's 2 cans of orange. And this one (points to the friend's batch), you have to subtract 1 can from the 4 (points to the 4 cans of water) to get the amount of orange.

Interviewer: Okay! I see just how you were thinking!