Lesson 1, Proportional Reasoning Unit
Instructor Notes

Preparation:

- Print copies of Activities 1 and 2 to distribute to groups.
- It may be useful to have a document camera for groups to share their drawings for Activity 2, Question 3.

Lesson:

1. **Introduce the Unit**

    - Follow Slides 3-5 of the Class PowerPoint

2. **Activity 1: Analyzing Student Thinking**

    - The goal of Activities 1 and 2 together is to explore the question, "What is proportional reasoning?", by looking at a student who knows and can correctly apply the proportion algorithm (Activity 1), but when given tasks that are slightly different from typical textbook problems (Activity 2), demonstrates a lack of proportional reasoning. The goal is to help problematize a notion that many preservice teachers come in with, namely that if you can correctly perform a procedure then you have understanding of that topic.
    - To set up Activity 1:
        - Introduce Bonita and the proportion algorithm of setting up a proportion and cross-multiplying (Slide 7)
        - Ask a class member to read a task that was given to Bonita in an interview with a researcher (Slide 8)
        - Present Bonita's work (Slide 8)
        - Distribute the Activity 1 worksheet to class members
        - Display Slide 9 and note that the task that Bonita worked on and the summary of her work are reproduced on the Activity 1 worksheet
        - Ask someone to read the two discussion questions aloud
    - Groups work.
    - Groups share. You can record responses using Slides 10 and 11

    - **Sample responses to Question 1**: If you were Bonita's teacher, what features of her work would you like?

- Bonita labeled the units, both when she set up the proportion (minutes and ounces) and in her answer (ounces)
- She showed her work of the cross-multiplication
- The proportion is mathematically correct even though minutes are in numerator
- Neat handwriting
- Correct answer

- **Sample responses to Question 2**: What would you ask Bonita next to see how strong her proportional reasoning is?

  - Use harder numbers, e.g., How long would it take for 10 ounces to drip?
  - What is the relationship between the 8 minutes and 6 ounces and the 4 minutes and 3 ounces?
  - What does same speed or rate mean? How many ounces dripped in 1 hour?

3. **Activity 2: Analyzing Student Thinking Continued**

- To set up Activity 2:
  - Tell your class: "Let's look at what tasks the researcher asked Bonia next"
  - Distribute the Activity 2 worksheet to class members
  - Slide 13: Ask a class member to read Task 2 aloud
  - Present the transcript on Slide 13 using role playing. Ask for a volunteer to be Bonita and a different volunteer to play the Interviewer
  - Slide 14: Ask a different class member to read Task 3
  - Have the volunteers playing Bonita and the Interviewer act out the interview. They can read either from Slide 14 or from the Activity 2 worksheet
  - Slide 15: Ask someone to read the discussion questions aloud. Point out that they have to use a **drawing** to SOLVE Task 3 and can't use an equation, graph, algorithm or procedure.
- Circulate as groups work. Question 3 is the most challenging, even for math majors:
  - You may need to prompt them that the drawing should be used to solve the task. In other words, it's not enough to solve the task by calculating and then illustrate the answer with a drawing. The solution should come from the drawing.
  - Additionally, if they say "multiply each number by 2" ask why that works: Can you show multiplication by 2 in a drawing?
  - If they appeal to multiplication by 2/2, which is 1, acknowledge that this will be especially helpful for secondary school students who see 2/2 as 1 and understand that any number multiplied by 2/2 is itself. For other kids, you may

want to create an argument that relies on what is happening in the context. Can anyone do that?

- Groups share. You can record responses using Slides 16, 17 and 19. You will need a board or document camera for groups to share their drawings for Question 3

- **Sample responses to Question 1**: Do you think Bonita is reasoning proportionally? Why or why not?

    - No, she's looking at how big the numbers are, not putting them into a proportion
    - No, because she doesn't know what she's really comparing – just looking at numbers without seeing a relationship between the 10 min and 3 oz. The drawings are also misleading, so that may be affecting her.
    - No, because she's not making a relationship between the 3 and the 10; and between the 6 and the 20

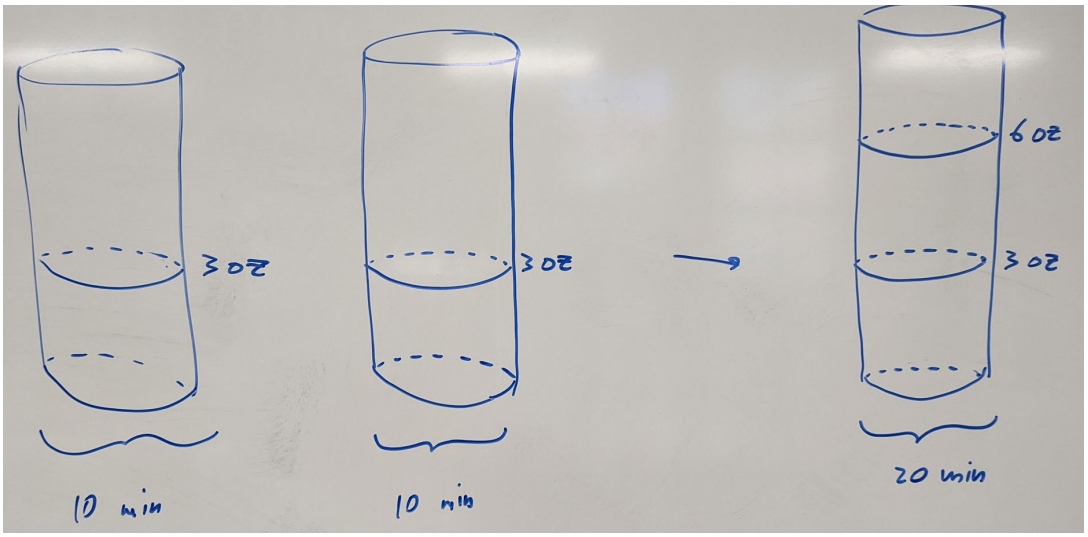
- **Sample responses to Question 2**: For Task 3, why do you think Bonita:
  a. thought that Crystal's faucet was dripping slower?

    - The amount of time for Crystal's faucet was greater, i.e., it will take more time for a certain amount of water to drip (20 min is longer than 10 min). Taking more time to drip means that faucet is slower. She didn't factor in the amount of water.

  b. thought that Crystal's faucet was dripping faster?

    - She saw more water in Crystal's glass – 6 oz versus 3 oz; Crystal has more water. Bonita didn't factor in time
    - Maybe she just looked at the numbers, 20 > 10 and 6 > 3; bigger is faster

- **Sample response to Question 3**: Solve Task 3 by drawing a picture and using reasoning only (no procedures)



Suppose Joanne's bucket is shown on the left. She collects 3 ounces in 10 minutes. Suppose she puts another bucket under her faucet and it also collects 3 ounce in 10 min. So the faucet is dripping at the same rate, since the same amount of water dripped in 10 minutes each time. If you put the two amounts together, you get 6 oz of water, which dripped in a total of 20 min. That's the same as what Crystal has. So the two faucets (Joanne's and Crystal's) are dripping equally fast.

- **Sample responses to Question 4**: Reflection on Bonita's work across the three tasks: What conclusions did you draw? What do you think is the point of Activities 1 and 2?
  - Even if we think students understand (after showing a procedure), don't assume they have the underlying understanding. They could be mimicking a procedure
  - Just because a student can compute, can perform a plug and chug, it doesn't necessarily mean they understand what they are doing
  - Students are often asked to be "one trick ponies" for the duration of one lesson – learn a procedure – but then when they move on they don't necessary know how to apply the procedure or know why the trick doesn't work elsewhere

4. **Homework 1**

- Follow Slide 20