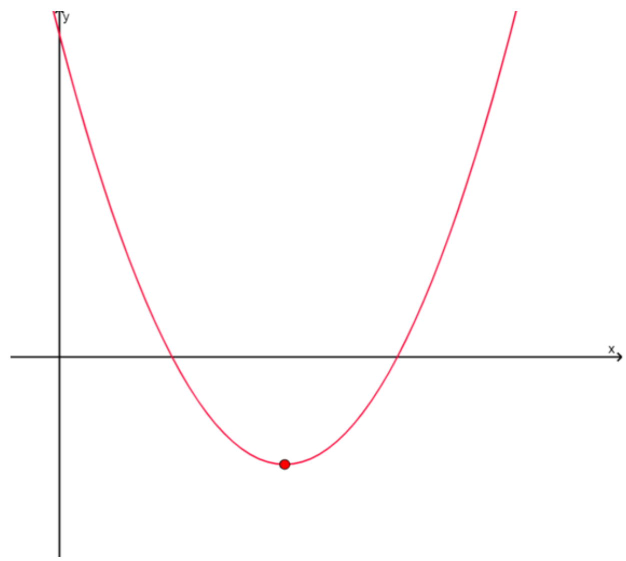
Lesson 6, Parabola Unit
Instructor Notes

Preparation

- Print copies of Activities 1 and 2 for class members
- A document camera will be useful for groups presenting their solutions to Activities 1 and 2
- Put the following image on a whiteboard or blackboard for use in the development of geometric meaning for the quadratic formula (see Item 3 of the Lesson below)



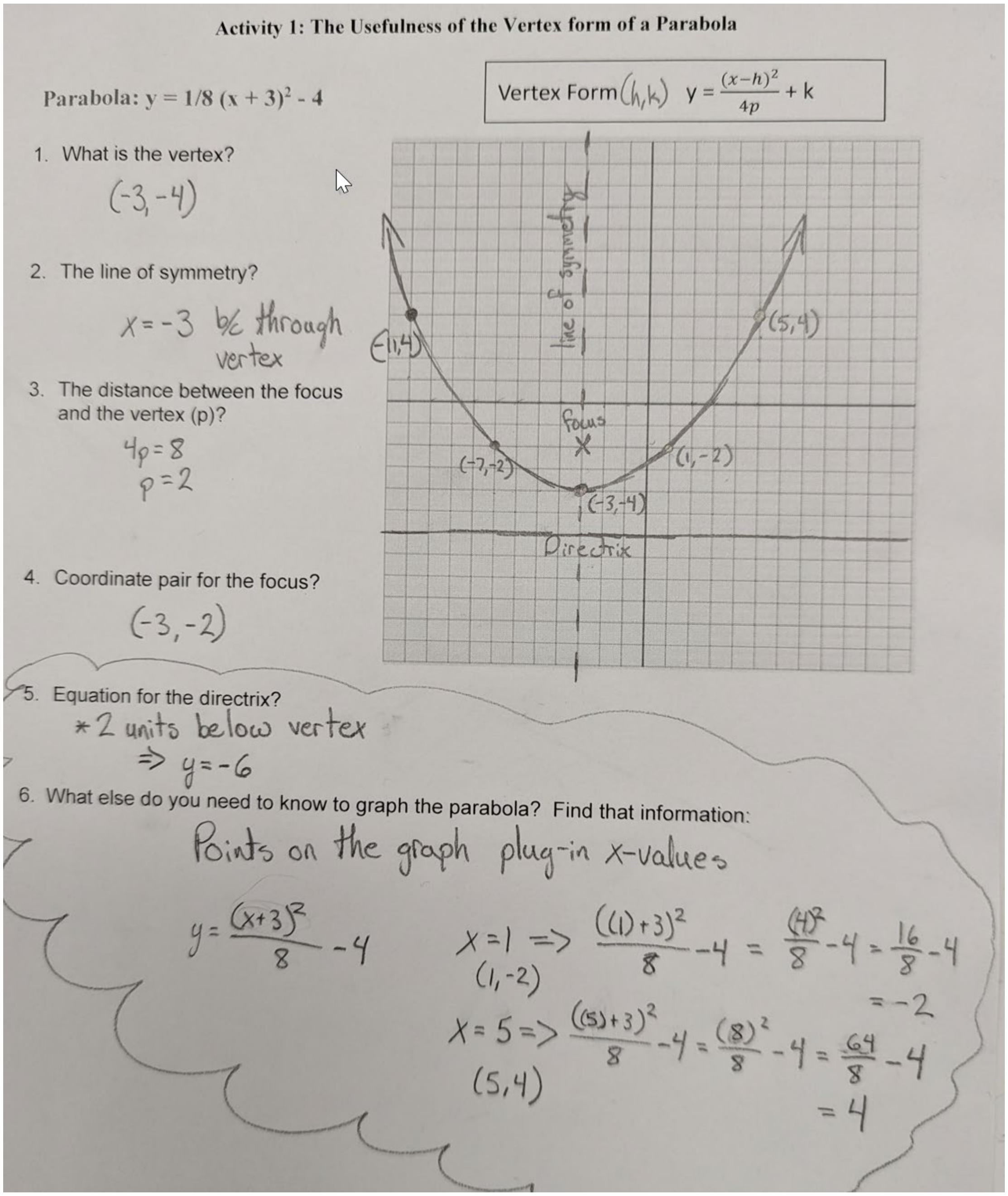
Lesson

1. Introduction to the Lesson

Follow Slides 2-3 of the Class PowerPoint

2. Activity 1: *The Usefulness of the Vertex form of a Parabola*

- Distribute the Activity 1 worksheets to class members
- You can set up the activity by using Slide 5 of the Class PowerPoint
- Circulate as groups work. This is a pretty straightforward task. The main mistakes are typically due to sign errors and arithmetic mistakes
- Choose a group to present; be sure they have split presentation among the group members so that everyone in the group has a chance to present something
- Sample response:



Activity 1: The Usefulness of the Vertex form of a Parabola

Parabola: $y = 1/8\,(x + 3)^2 - 4$

Vertex Form (h,k) $y = \frac{(x-h)^2}{4p} + k$

1. What is the vertex?

$(-3,-4)$

2. The line of symmetry?

$x=-3$ b/c through vertex

3. The distance between the focus and the vertex (p)?

$4p=8$

$p=2$

4. Coordinate pair for the focus?

$(-3,-2)$

5. Equation for the directrix?

* 2 units below vertex

$\Rightarrow y=-6$

6. What else do you need to know to graph the parabola? Find that information:

Points on the graph plug-in x-values

$y=\frac{(x+3)^2}{8}-4$

$x=1 \Rightarrow \frac{((1)+3)^2}{8}-4 = \frac{(4)^2}{8}-4 = \frac{16}{8}-4 = -2$ $(1,-2)$

$x=5 \Rightarrow \frac{((5)+3)^2}{8}-4 = \frac{(8)^2}{8}-4 = \frac{64}{8}-4 = 4$ $(5,4)$

3. How to Re-express an Equation from Standard Form to Vertex Form, Using Informal Reasoning

- **Slide 8:** This slide sets the goal and points out that we'll be using informal reasoning in this lesson, rather than a procedure or formula.
- **Slide 9:** Work through this slide interactively with the class. Target Responses:

- Question 1: $y = x^2 + 6x + 9$
- Question 2: $y = (x + 3)^2 + 0$; or $y = (x + 3)^2$
- Question 3: We subtracted 1 from the original given equation to get the close equation

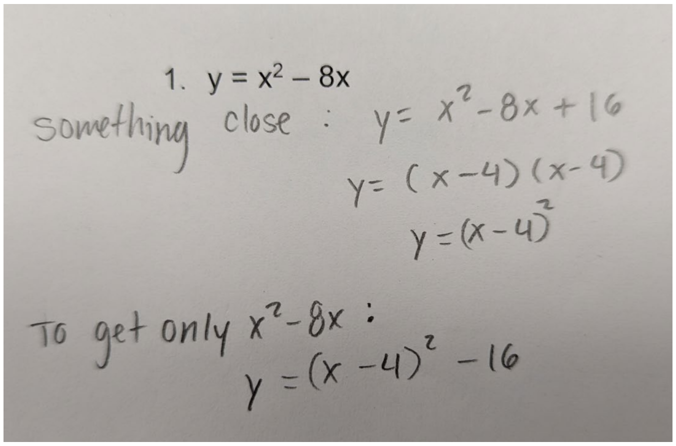
- **Slide 10:**
  - Point out that since they subtracted 1 from the original equation, $y = x^2 + 6x + 10$ to get $y = x^2 + 6x + 9$, then $x^2 + 6x + 10 = (x^2 + 6x + 9) + 1$
  - Answer to question on the slide: The vertex is (-3, 1)

4. Activity 2

- Distribute the Activity 2 worksheets to class members
- You can set up the activity by using Slide 12 of the Class PowerPoint. Remind students to use informal reasoning, like we did in the item we worked on as a class, rather than a procedure or rule that they were taught. Part of the reason for this is to be able to connect with the reasoning of high school students who have not yet been exposed to a rule and to be able to have some way to unpack the rule some of you were taught.
- Even though all groups should work on all three questions for Activity 2, it helps to assign each group to one of the items to present.
- Visit groups. A common problem is for math majors to remember a rule for completing a square but not be able to unpack that rule. To probe, ask:
  - For Task 1, where does the 16 come from?
  - For Task 2, how do you know to use 1 in the "close equation"
- Answers:
  1. $y = (x-4)^2 – 16$; Vertex is (4, -16); $p = ¼$
  2. $y = (x-1)^2 – 6$; vertex is (1, -6)
  3. $y = 2(x+5)^2 + 16$; vertex is (-5, 16)

     If they have $y = 2((x+5)^2) + 8)$, ask what vertex is?

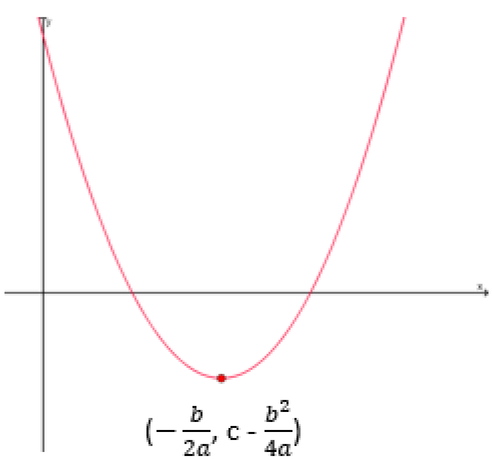
     Sample Student Work for Question 1:



5. Geometric Meaning for the Quadratic Formula (Whole Class)

- The goal of Slides 14 – 20 is to help develop geometric meaning for a formula that math majors know very well (but often only procedurally)
- This section is an interactive class discussion
- In this section, you will be asking class members to make additions to the drawing on board – see Preparation
- **Slide 15** – The goal of this slide is to give pre-service teachers a quadratic function written in standard form the equivalent vertex form, instead of taking class time to complete the square.
- **Slide 16**: What is the vertex of the parabola?
  Answer (by getting the information from the vertex form of the general parabola): $(-\frac{b}{2a}, c - \frac{b^2}{4a})$

  Ask someone to label the vertex for the general parabola on the board, e.g.:



- **Slide 17:** What is a root**?**

  Sample responses:
    - x-intercept
    - the x value of the function when y = 0
    - to find a root, use the quadratic formula
    - the roots are symmetric with respect the vertex
    - solution to the equation
    - a zero [note that this student didn't stipulate whether it was y or x that is equal to zero]

  You can see from these responses that even math majors can struggle with the definition of a root.

  Tell then that conventionally, a root is the x-value when y is 0. So, a root is the x-value of an x-intercept. A root is not identical to an x-intercept

- **Slide 18:** Roots of a quadratic function

  The goal of this slide is to give pre-service teachers is to provide the result of solving for x when y = 0 in $y = a(x + \frac{b}{2a})^2 + c - \frac{b^2}{4a}$, rather than taking the time to perform the algebraic manipulations. That way you can focus on the meaning of the quadratic formula in the next slide.

- **Slide 19:** Geometric Meaning of the Quadratic formula

  - **Question 1**: What does $-\frac{b}{2a}$ tell you about the graph?

    Answer: the equation x = -b/2a is the line of symmetry

    Ask a class member to drawn and label the line of symmetry on the graph on the board

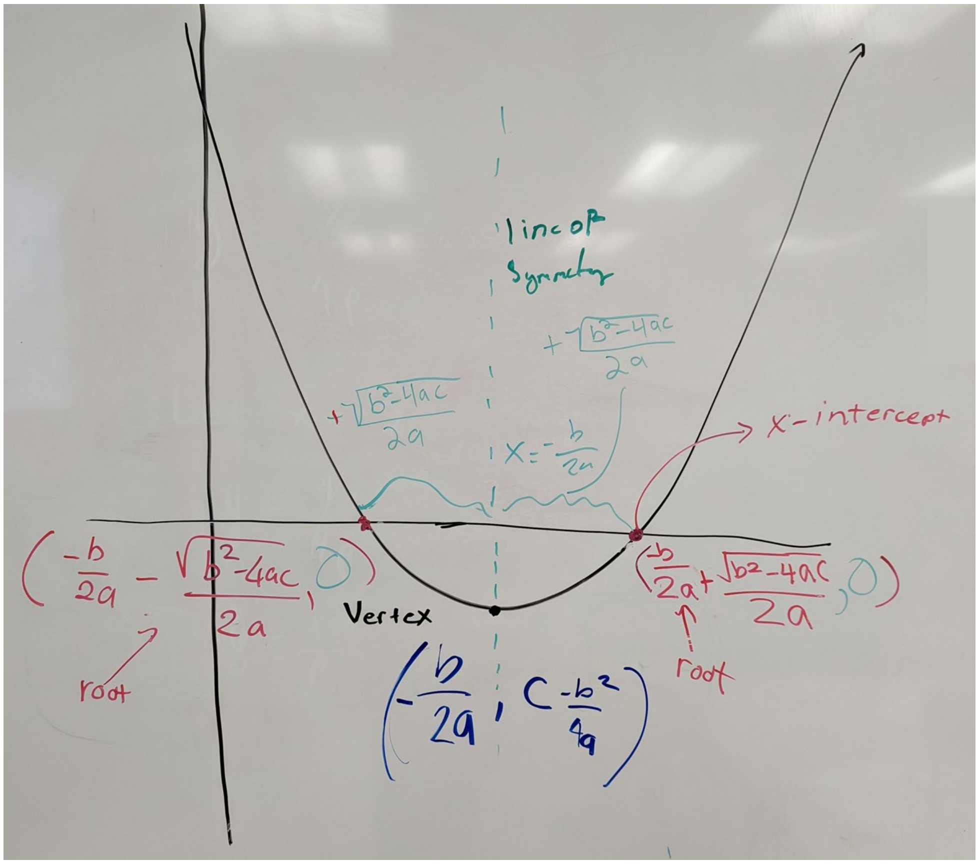
  - **Question 2:** What does $\frac{\sqrt{b^2-4ac}}{2a}$ tell you about the graph?

    Sample response: the distance from the line of symmetry to each root

    Ask a class member to label this distance on the graph

- **Question 3:** Label the coordinate pairs that contain each root.

  Answer: $(\frac{-b}{2a} + \frac{\sqrt{b^2-4ac}}{2a}, 0)$ and $(\frac{-b}{2a} - \frac{\sqrt{b^2-4ac}}{2a}, 0)$



- **Slide 20:** Geometric Meaning of the Quadratic formula

  Ask several students to interpret the quadratic formula geometrically – put what they have learned today in their own words.

6. Homework 6

- Follow Slide 21