Lesson 5, Parabola Unit
Instructor Notes

Preparation

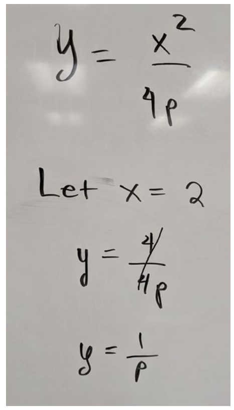
- Print copies of the Activity 2 and 3 worksheets to distribute to groups. If you decide to use optional Activities 1 and 4, also print those worksheets.
- It will be helpful to have a document camera for class members to share their work.
- Be prepared to leave the Class PowerPoint and demonstrate the following GeoGebra applet: https://www.geogebra.org/m/myQ4WTtx

Lesson

1. Activity 1: Review & Extend (Optional)

- The goal of this activity is to allow groups a chance to solidify their argument in response to the Parameter Change Task from Lesson 4. This is a challenging task and often takes math majors more than one class period to find a satisfying approach.
- Whether or not you use this activity depends upon how far groups got on the Parameter Change Task in Lesson 4.
- Distribute the Activity 1 worksheets to class members
- You can set up the activity by using Slide 3 of the Class Powerpoint
- For Activity 1, let groups decide what makes the most sense for their group: (a) complete their approach from the previous class; (b) revoice one of Sasha and Keoni's arguments, which they viewed for Homework 4; or (c) pursue a new explanation.
- Circulate and identify any new approaches or ideas that you'd like to highlight for the whole class.
- Either during the time that groups work, or during class presentation, prompt students who have limited their argument to purely a numeric one to connect their argument to the quantity of distance on the coordinate plane.

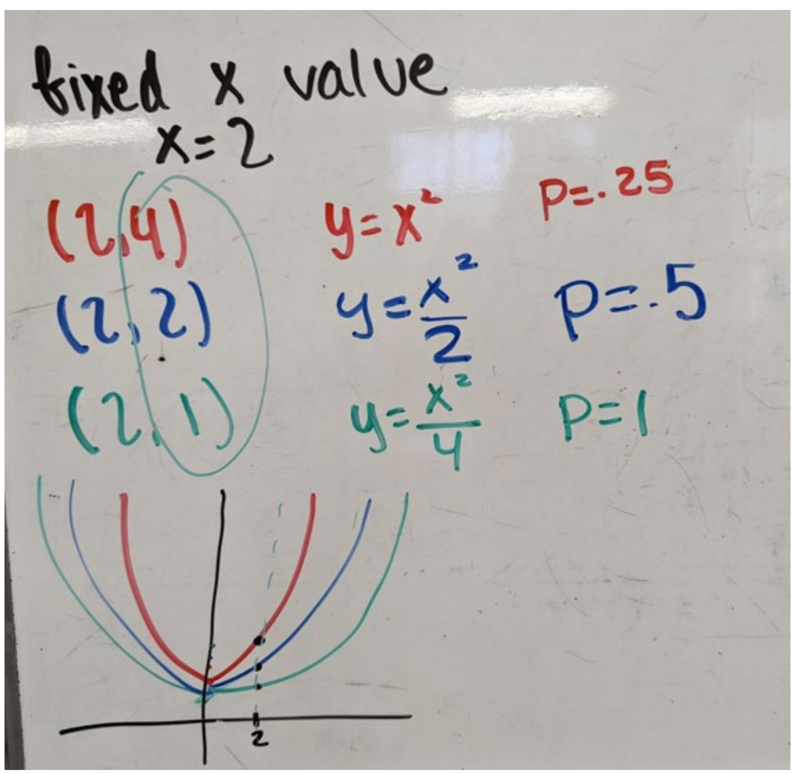
  For example, suppose a group substitutes 2 for x into the equation $y = x^2/4p$, and then simplifies the equation to get $y = 1/p$:



They then argues that as p increases the fraction 1/p (which is y) decreases. That means as p increases from ¼ to ½ to 1, the y-values decrease from 4 to 2 to 1.

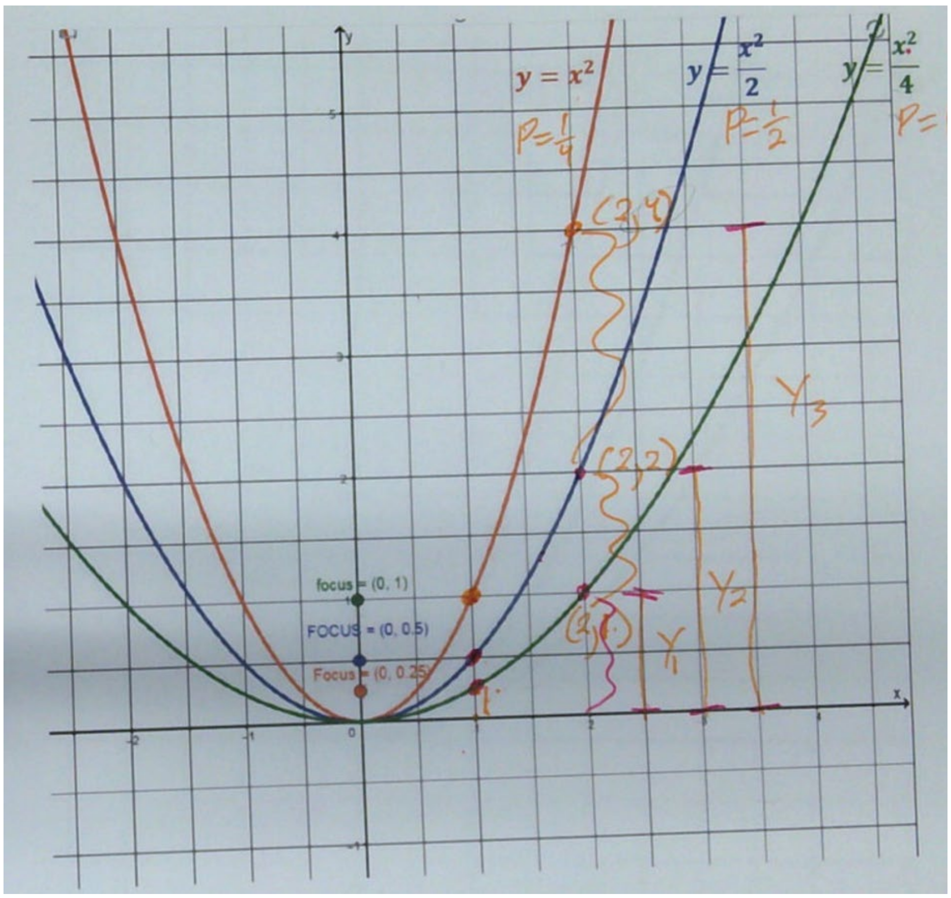
A good follow-up is to ask, “That’s a really important insight. Now, can you take it one step further and refer to your graphs to show why the y-values decreasing for a fixed x-value of 2 force the parabola to get wider?”

Suppose the class responds with the following graph and gesture to the decreasing values of 4, 2, and 1 for the y-values as p increases.



To help students connect this numeric pattern to distances on the graph, you could ask, “That’s an important numeric pattern! Now the relationship that you and Keoni and Sasha noticed is a geometric and visual pattern, namely that the parabola gets wider as p increases. Is there anything visual on the graph that you can point to that helps explain why the parabola appears to get wider?”

A good response would be the following:



Each y-value of a point (2, y) on the parabola represents the distance from the x-axis to that point (2, y). Look at the 3 distances y_3, y_2 and y_1 labeled on the graph. So as y decreases (but x = 2), the distance between the point (2, y) parabola is forced to move closer and closer to the x-axis, which has the effect of “pulling down” the parabola closer and closer to the x-axis, which has the visual effect of widening the parabola.

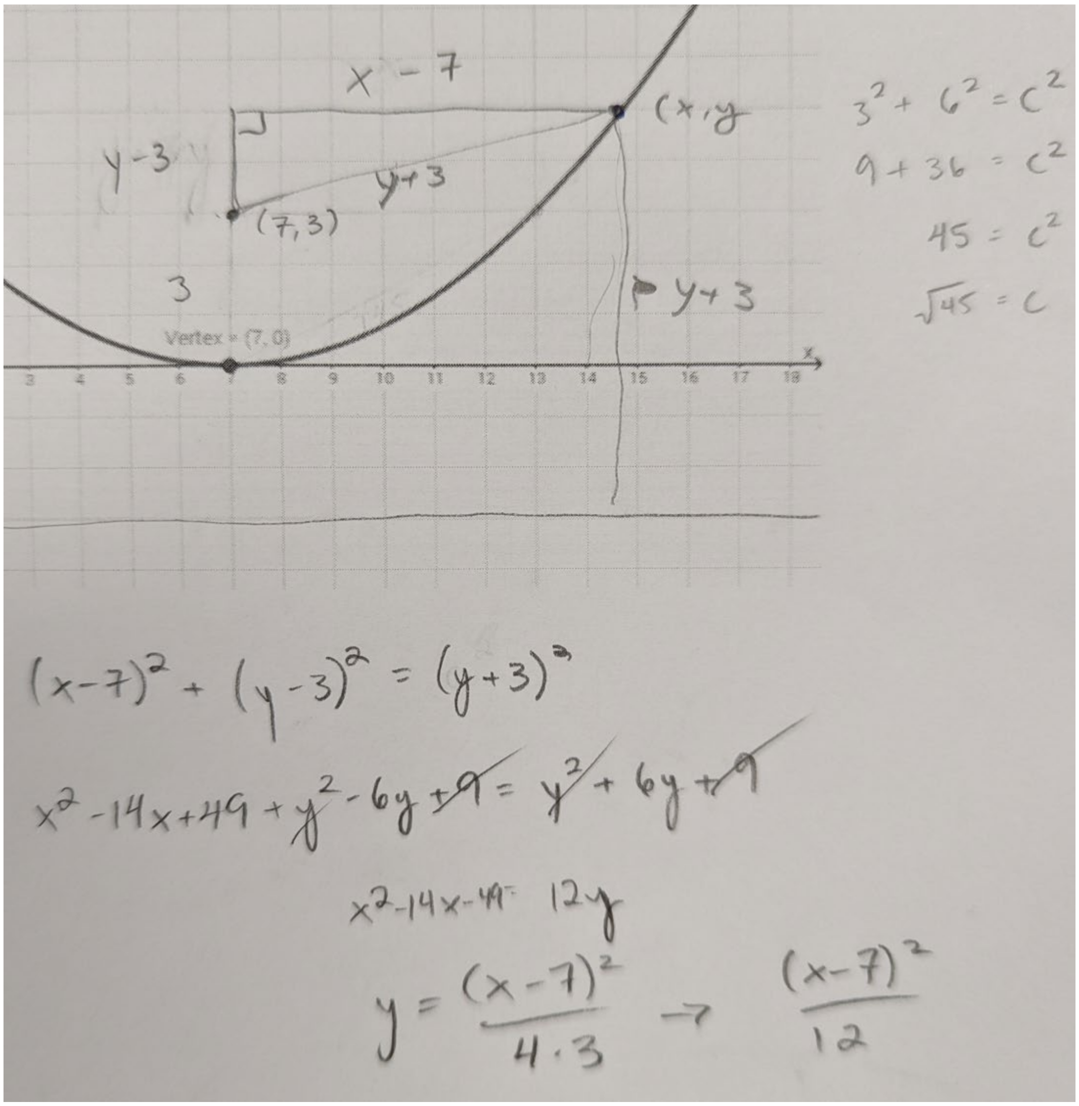
2. Introduce the Theme of Developing the Vertex Form of the Equation for a General Parabola

- Follow **Slides 5-10** of the Class PowerPoint
- **Slide 7** Sample responses
  - Derive equations for parabolas with different vertices, like (2, 0)
  - Do similarly to their previous activities – use specific examples, like leave p the same, e.g., p = 3, use vertex (0, 0) and (2, 0). Want them to see that the points are the same but shifted to the right.
  - Help them see the (h,k) translates the parabola – could use lines to help.
  - Use Desmos; play around with it and move the vertex around and see what they notice
- **Slide 8: Demonstrate an Applet**
  - Leave the PowerPoint and go to: https://www.geogebra.org/m/myQ4WTtx
  - Move each slider and ask your class what they think high school students will notice?

3. Activity 2: Derive the Equation of a Parabola with p = 3 and Vertex (7, 0)

- Distribute the Activity 2 worksheets to class members
- You can set up the activity by using Slide 12 of the class Powerpoint
- Circulate as groups work. Watch for three challenges that math majors sometimes experience:
  - They may have trouble getting started, because the graph looks different than those they have tackled in previous lessons, namely the focus is not labeled. Additionally, they may not have noticed that p = 3 in the task statement. You can ask, "Do you have enough information to label the focus of this parabola?"
  - Sometimes groups label a point on the parabola that is aligned horizontally with the focus (7, 3), instead of using a general point. Then they have trouble setting up the right triangle. You can mention that the point they used, (13, 3), is a "special point" which has special properties, since it is aligned horizontally with the focus. Instead, to create an equation for the parabola, which is a statement of the relationship between all x and y values on the parabola, they want to identify a more general point (x, y) on the parabola.
  - Some class members may think that the horizontal leg of the right triangle that they need to create for their derivation will have a length of x, just as it has for previous parabolas that they have worked with in this class. But the distance should be x – 7. You could ask them to point with their finger where the distance associated with the x in their general point (x, y) is on the graph.

- Select one or more groups to share. Alternatively, you can skip sharing, watch the next video where Sasha and Keoni solve the same task, and then compare their work with the students in the video.
- Sample solution:



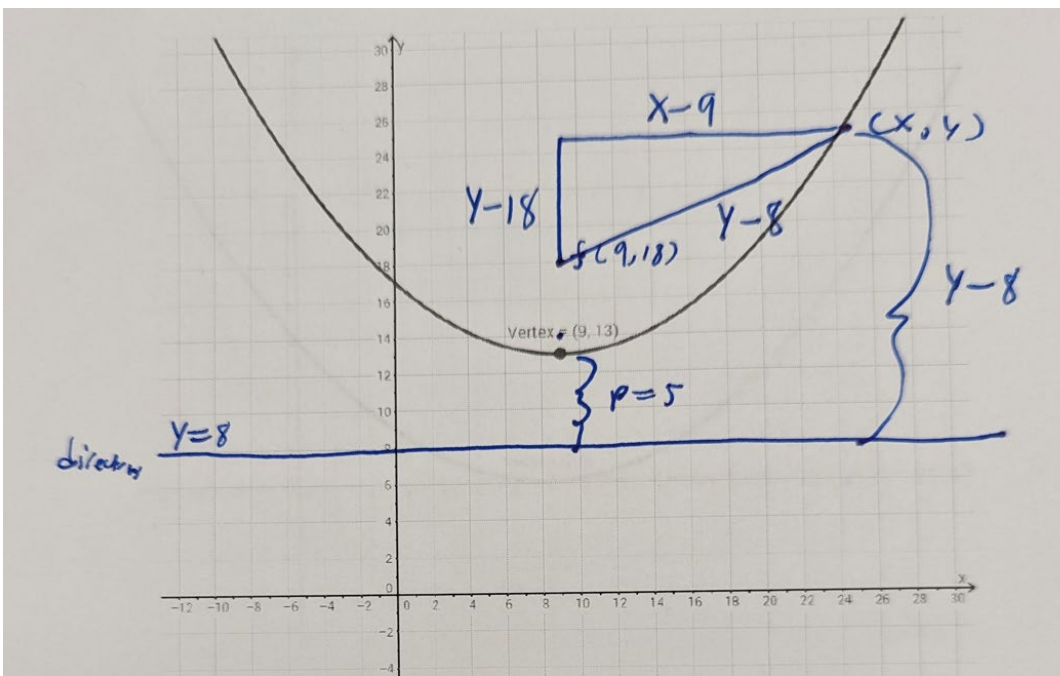
4. Watch and Discuss Video 1

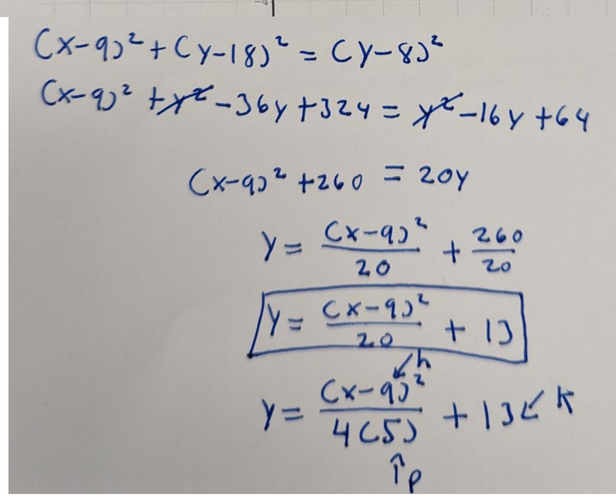
- Follow Slides 14 – 17 of the Class PowerPoint

5. Activity 3: Derive the Equation of a Parabola with p = 5 and Vertex (9, 13)

- Distribute the Activity 3 worksheets to class members

- You can set up the activity by using Slide 19 of the Class PowerPoint.
- As you circulate while groups work, be aware that there are three main points of difficulty: (a) groups sometimes use p = 3 instead of p = 5; (b) the fact that the directrix is between the vertex and the x-axis means that the distance between the point (x, y) and the directrix will involve subtraction (rather than addition, as it has for previous parabolas); and (c) finding the distance of the vertical leg of the right triangle is more complex because of the adjustment of the parabola's vertex above the x-axis.
- Select one or more groups to share. Alternatively, you can skip sharing, watch the next video where Sasha and Keoni solve the same task, and then compare their work with the students in the video.
- Sample solution:





6. **Watch and Discuss Video 2**

- Follow Slides 21 - 24 of the Class PowerPoint

7. **Optional Activity 4**

- The task for Activity 4 is similar to Activity 3, in that the vertex for both parabolas is (9, 13). However, the task for Activity 4 uses an unknown p-value rather than a p-value of 5. This makes the derivation more challenging, because one needs trinomials to express the distance of the general point to the directrix and the length of the vertical leg of the right triangle.
- Activity 4 provides a good transition to the general derivation of the vertex form of the equation for a parabola in Homework 5

8. **Homework 5**

- Follow Slide 25