Lesson 4, Parabola Unit
Instructor Notes

Preparation

- Print copies of Activity 1 and Activity 2 to distribute to groups.
- It will be helpful to have a document camera for class members to share their work on Activities 1-2 with the whole class.
- It's important that class members have completed Homework 3 prior to this lesson.

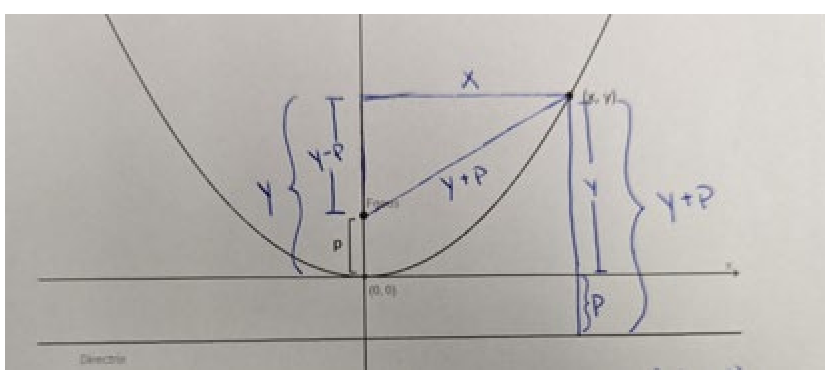
Lesson

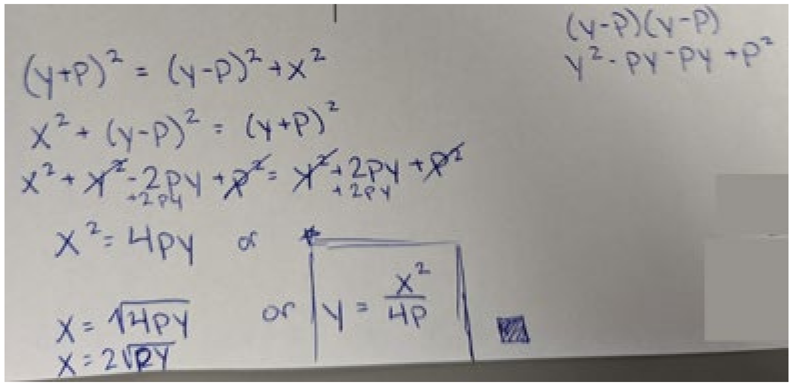
1. **Introduction to the Lesson: The Power of Generalizing**

   - Follow Slides 2-4 of the Class PowerPoint

2. **Activity 1: Derive the Equation of a General Parabola with Vertex at the Origin**

   - Distribute Activity 1 worksheets to groups
   - Display Slide 6 of the Class PowerPoint. Ask someone to read the top part, just through the equations. Then advance the animation to highlight the introduction of the letter p. Make sure class members see the meaning of p in the graph as the distance between the vertex and the focus. Then complete the reading of the task.
   - Circulate as groups work. Below is a sample solution by math majors:





- Select a group to present their solution. Then see if any other groups have differences in their approach that they would like to share
- Let class members know that they will watch Sasha and Keoni do Activity 1 as part of Homework 4.
- After groups have shared, follow Slides 8 and 9 to highlight the meaning of y + p and y – p from the derivation, as distances.
  - For **Slide 8**, be sure to elicit two interpretations for y + p: (a) as the process of combining the distance from the general point (x, y) to the x-axis with the distance between the x-axis and the directrix; and (b) as an entity, namely the distance from the general point (x, y) to the directrix
  - Similarly for **Slide 9**, evoke two interpretations for y – p: (a) as the process of starting with the distance from (0, y) to the origin and then taking away the distance between the origin and the focus; and (b) as an entity, namely the distance between the point (0, y) and the focus (0, p)
- Discuss the meaning of the equation y = $\frac{x^2}{4p}$, using **Slide 10**. Sample Responses:
  - Class members who perceive the equation as representing a single parabola may say, “The relationship between x and y values correspond to the parabola,” or “A representation of the set of points on a parabola with p distance between the focus and the vertex”
  - If you get these types of responses, press by asking, “Does the equation represent a single parabola or more than one parabola?”
  - Ultimately you are looking for a response like the following: This equation represents the family or collection of parabolas that have a vertex at the origin (and that open up)

3. Introduce Parameters

- Follow Slides 11-13

4. **Activity 2: Parameter Change**

- **Slide 14.** The purpose of this slide is to start by establishing THAT there is a relationship between increasing (or decreasing) the value of p and the apparent shape of the parabola. This will make it easier, when you get to Activity 2 to distinguish the relationship from EXPLAINING WHY the relationship holds. Sample responses to the question on Slide 14:
  - If p increases then the parabola looks like it is getting wider
  - If p decreases, the parabola gets narrower

- **Slides 15 -16.** Watch a 45 sec video of Sasha and Keoni stating the relationship that they see.

- **Slide 18.** Distribute the Activity 2 worksheets to class members and use Slide 18 to set up the task. Because students seem to sometimes only want to engage with this task in a superficial and quick manner, it's important to let them know that it is challenging and that it's not enough to just state an informal intuition (e.g., that the parabola is growing faster when p decreases, which makes it steeper). Instead, they need to find particular points on each parabola (by using the three equations) and link their intuition to those points.
- Circulate as groups work.
- If you see an argument like the following,

Everytime we double our P, our y is getting divided by 2.

$\frac{x^2}{4p}$

$\frac{x^2}{4(\text{big})} = \text{small}$

$\frac{x^2}{4(\text{small})} = \text{big}$

Ask them:

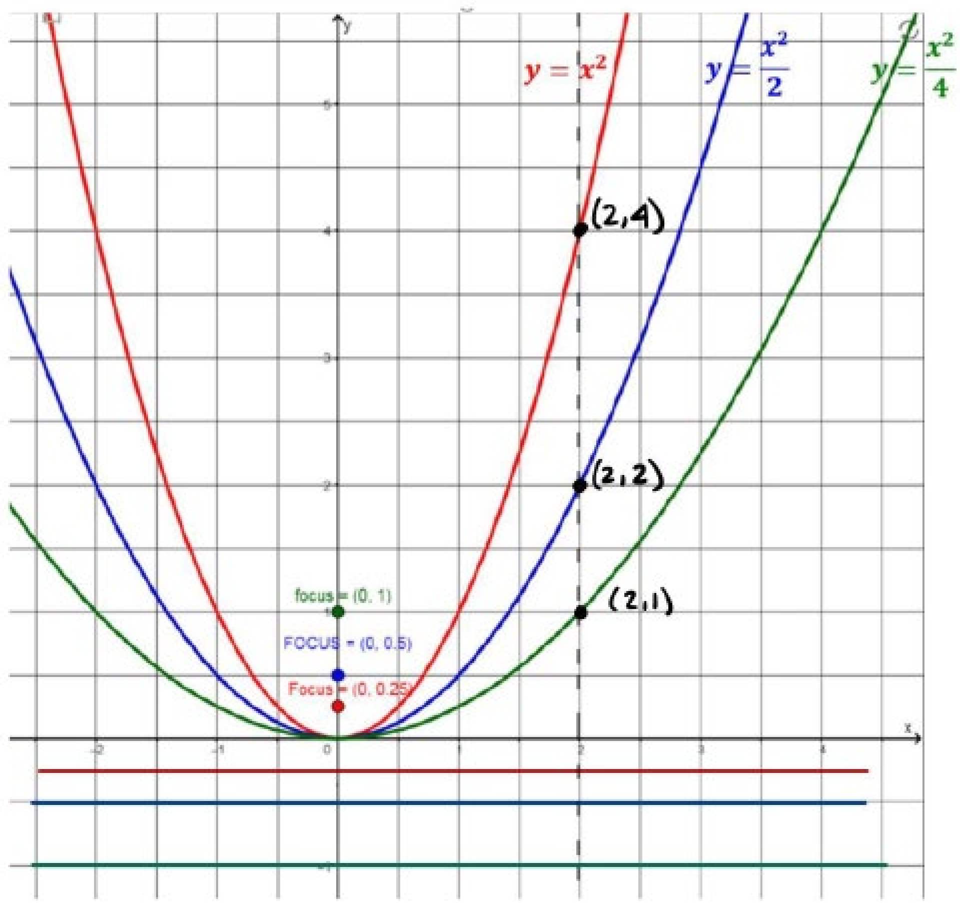
- if p increases, how to you know that x^2 will be smaller than 4p; can't I select an x so that the numerator is actually larger than the denominator? For example, pick a large value for p, such as p = 100. Suppose I then let x = 1000, will $x^2/4p$ be large or small? (2500)
- What would need to be true for your argument to work?
- Can you use the graphs?

- If you see an argument like the following:

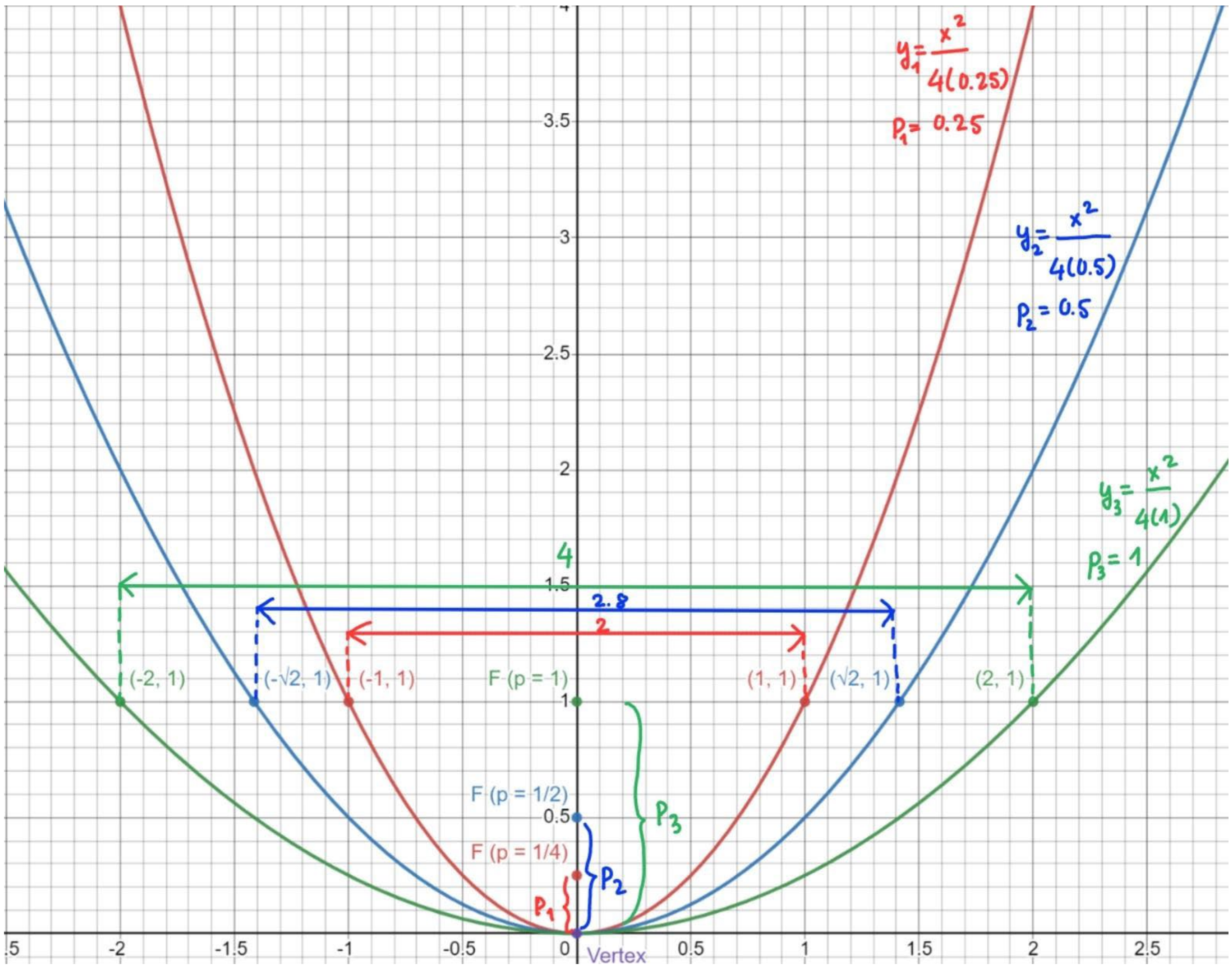
$\lim_{p \to \infty} \frac{x^2}{4p} = 0$ As p increases indefinitely, the parabola will straighten out to a straight line.

Remind them to only use mathematical tools available to a typical high school student (say, a student in Grade 10). Encourage them to identify points on each of the graphs and use them in their argument.

- To locate groups that you may want to have share their progress with the whole class, watch for groups who fix the x value of the three graphs, e.g., let x = 2 and identify the points (2,1), (2,2) and (2,4):



Or who fix the y-value (e.g., let y = 1) and identify the points (1, 1), ($\sqrt{2}$, 1), and (2, 1).



- Once either an x-value or a y-value is fixed, then it is easier to get a handle on what is happening to the shape of the graph as p increases.
- You may want to stop the groups working, have one or two groups share what they have discovered, even if complete arguments are not yet done. Then have groups resume.
- If you happen to have a group finish early, ask them to create a second explanation using a different approach.
- Here's a sample argument that a student made using the second graph, where y was fixed as 1:

To see how the width of the parabola changes as p increases, let's look at the points on each parabola where $y = 1$.
For the first parabola, the points are (1, 1) and (-1, 1). The distance between these two points is $1 + 1 = 2$.
For the second parabola, the points are ($\sqrt{2}$, 1) and ($-\sqrt{2}$, 1). The distance between these two points is $\sqrt{2} + \sqrt{2} = 2\sqrt{2} \cong 2.8$.
For the third parabola, the points are (2, 1) and (-2, 1). The distance between these two points is $2 + 2 = 4$.

We can see that as p increases (i.e., we move from the red parabola to the blue parabola to the green parabola), the distance between the points where $y = 1$ becomes more spread out ($1 < 2.8 < 4$). This means that the parabola appears wider.

5. Homework 4

Slide 20: Set up Homework 4 by letting class members know that Sasha and Keoni actually explored three different explanations for why the parabola appears to get wider as p increases. One explanation uses a fixed x-value of 2. A second explanation uses a fixed y-value of 4. The last explanation uses what they call "special points." These are the points on a parabola that are horizontally aligned with the focus. In Homework 4, your students will view two of these explanations (the one using a fixed x-value and the "special points" argument) and put Sasha and Keoni's explanations in their own words (and with more sophisticated adult language). [Note that the argument where y is fixed is a good candidate to use on an exam.] Thus, it's OK if students leave the class not being sure of a solution to Activity 2, because they will have the opportunity to view videos of Sasha and Keoni working on the same task.