Lesson 3, Parabola Unit
Instructor Notes

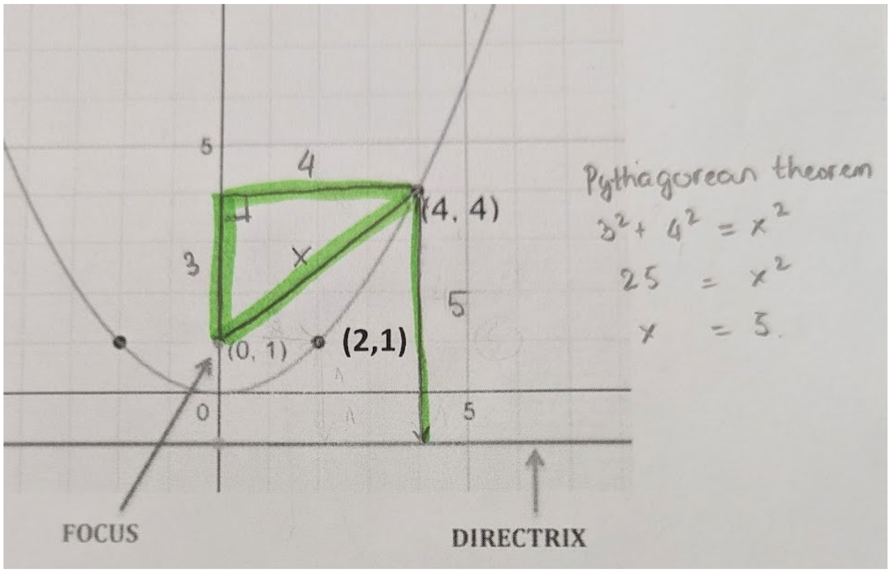
Preparation:

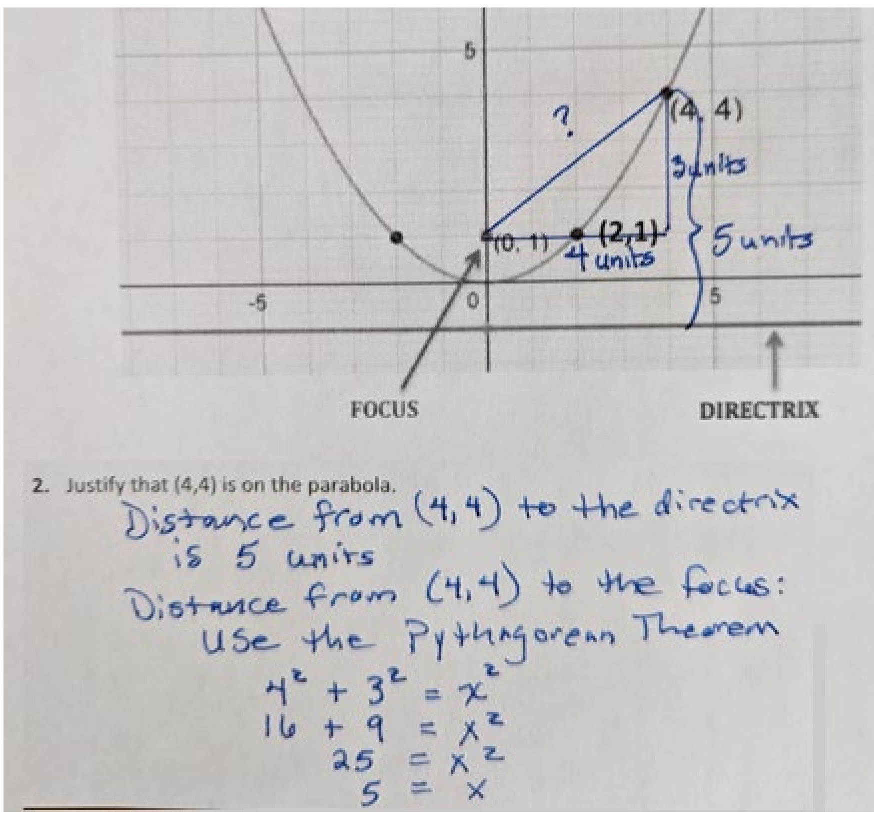
- Print copies of Activity 1, Activity 2, and Activity 3 to distribute to groups.
- It will be helpful to have a document camera for class members to share their work on Activities 1-3 with the whole class.

Lesson

1. **Introduction to the lesson: Connecting Geometry with Algebra**
   - Follow PowerPoint Slides 2 and 3

2. **Activity 1: Justifying Points on a Parabola**
   - Distribute Activity 1 worksheets to groups
   - You can display Activity 1 – see Slide 5 of the Class PowerPoint – and ask someone to read the task aloud. Highlight that they need to use the geometric definition of a parabola in their justifications.
   - Circulate as groups work.
   - Question 1 can be answered easily by using the grid to count the number of units from the point (2,1) to the focus (namely, 2 units), and the number of units from the point (2,1) to the directrix (again, 2 units). Because these distances are the same, the point satisfies the geometric definition of a parabola, and thus are on the parabola.
   - You may notice your students having more difficulty justifying that (4, 4) is on the parabola. That's because they will need to use additional mathematical tools, such as the Pythagorean Theorem or the distance formula. Possible questions you can ask such students:
     - What are you trying to find?
     - What distances do you already know?
     - To find the distance from (4, 4) to the focus, as there any other math tools that you could bring to bear?
   - Ask group presenters to share with the whole class. Note that there are different possible solution methods for Question 2. The two solutions shown below both use the Pythagorean Theorem, but they use different placements of a right triangle:





3. **View and Discuss Videos 1 and 2 (High School Students Solving the Tasks from Activity 1)**
   - Follow Slides 7-10
   - Sample Responses to the Questions on **Slide 10**:
     Question 1: How did your group's work on this task compare with Sasha and Keoni's:

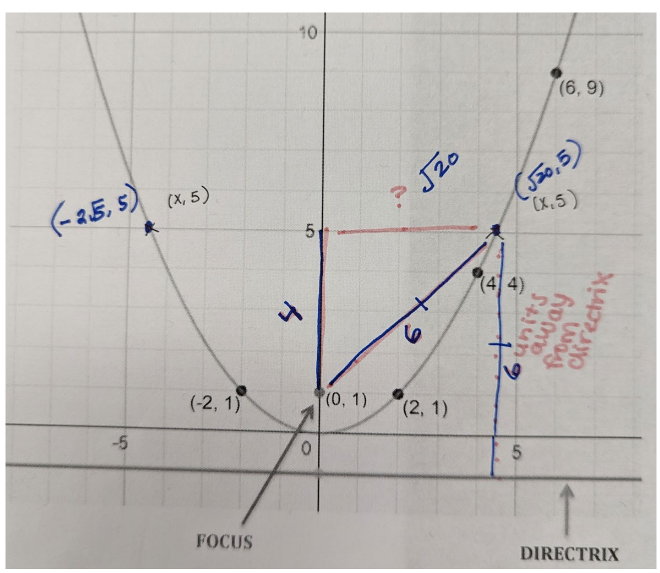
- Class members may compare the orientation and placement of their right triangle with Sasha and Keoni's
- Some class members may have used the distance formula, rather than the Pythagorean Theorem, which provides an opening to compare the two

Question 2: Anything else you noticed in the video that you want to share?

- Sasha and Keoni identified the a and b values of 3 and 4 without realizing that it would help them for the Pythagorean Theorem
- I wonder if they would have seen the triangle more quickly if they had drawn straight lines rather than the "hops"
- It seemed to help Sasha and Keoni when the teacher asked them what they knew and what they were trying to find

4. Activity 2: Given a y value, find x

- Distribute Activity 2 worksheets to groups
- You can display Activity 2 – see Slide 12 of the Class PowerPoint – and ask someone to read the task aloud.
- Circulate while groups work. You may need to remind class members to make predictions about Sasha and Keoni (Question 2) and not just stop after Question 1. You can also select who you would like to share their solution to Question 1.
- Sample response to Question 1: Find the x-value when the y-value is 5.



- First we drew a line at y = 5, since the point we are interested in will be where that line intersects with the parabola.

- Then we draw a line segment from the targeted point (x, 5) to the directrix. The distance is 6 units.
- Because of the definition of a parabola, we know that the distance between (x, 5) and the focus will also be 6.
- We make a right triangle and count the distance from the focus, (0,1), to the line at y= 5, which is 4 units
- We can then use the Pythagorean Theorem to find the distance of the horizontal leg of the right triangle, which also gives us the x-value:

$$4^2 + x^2 = 6^2$$
$$16 + x^2 = 36$$
$$x^2 = 20$$
$$x = \sqrt{20}$$

- Sample responses to Question 2: Make predictions about Sasha & Keoni.

a. What do you think they can understand that they can use?
   - Sasha and Keoni will realize that the hypotenuse of their right triangle is the same distance as the distance from the point to the directrix

b. What might they have trouble with?
   - Deciding what side to start on – positive or negative
   - They might forget that the distance from the point to the directrix is the same as the distance from the point to the focus
   - They might mess up putting the 6 in the wrong place, because usually with the Pythagorean theorem you are given the sides and find the hypotenuse
   - May not square the 6
   - Will they get both points?

5. View and Discuss Video 3

- Follow Slides 15-17 from the Class PowerPoint
- Sample Responses to the Questions on Slide 17:

Question 1
- Surprised they did so well!
- I did think they'd use the definition of a parabola, which they did
- They almost didn't square the 6
- They didn't consider the negative

Question 2

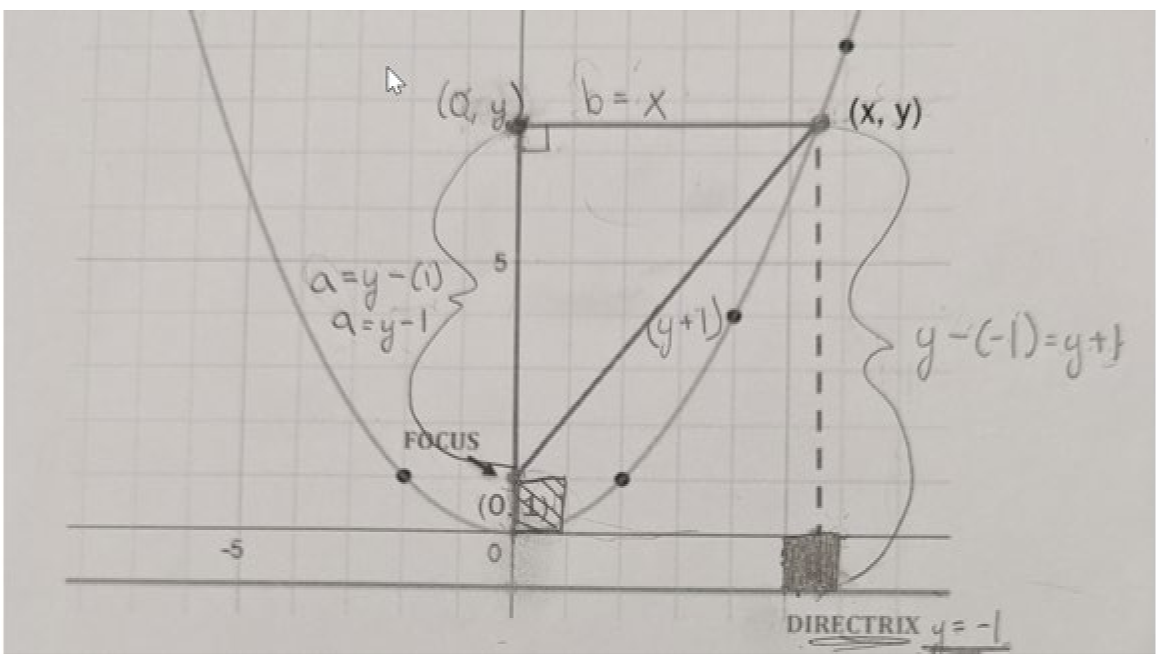
– They took to heart "We proved this before, so we can keep using it"

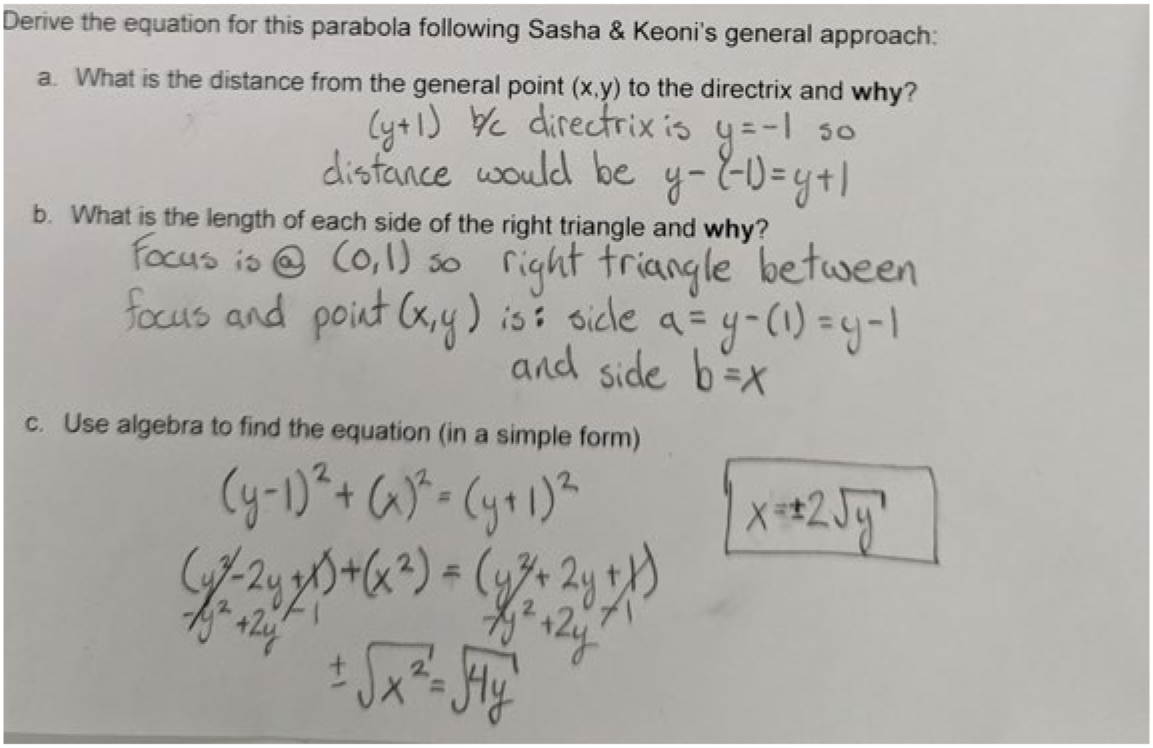
6. Generalizing a Method

- Follow Slides 18-20 of the Class PowerPoint

7. Activity 3: Derive the equation for a parabola

- Distribute Activity 3 worksheets to groups
- You can display Activity 3 – see Slide 22 of the Class PowerPoint – and ask someone to read the task aloud.
- Circulate while groups work. If a group is struggling, you may want to ask them how they would approach the task if they knew the value of y, e.g., if y = 12. Then generalize for an unknown y-value
- Some groups may struggle with expressing the distance from the point (x, y) to the directrix as y + 1, and the distance from the focus to the horizontal leg of the right triangle as y – 1.
- Sample response to Activity 3:




Derive the equation for this parabola following Sasha & Keoni's general approach:

a. What is the distance from the general point (x,y) to the directrix and **why**?

(y+1) b/c directrix is y=-1 so distance would be y-(-1)=y+1

b. What is the length of each side of the right triangle and **why**?

Focus is @ (0,1) so right triangle between focus and point (x,y) is: side a = y-(1) = y-1 and side b=x

c. Use algebra to find the equation (in a simple form)

$(y-1)^2+(x)^2=(y+1)^2$

$(y^2-2y+1)+(x^2)=(y^2+2y+1)$

$-y^2+2y-1 \quad -y^2+2y-1$

$\pm\sqrt{x^2}=\sqrt{4y}$

$x=\pm 2\sqrt{y}$

- Sharing. You may want to start by asking one group to share their response to all parts of Activity 3. Then see if any groups have different explanations for Parts a and b, and different versions of the equation for Part c.
- Reflect. Follow Slides 24 – 26
  - The purpose of Slides 24 and 25 is to highlight the meaning of y + 1 and y – 1 in terms of distances on the graph.
  - Slide 26:
    - You may want to ask, "Why do you think Sasha and Keoni use the letter b (instead of x)?"
    - You may want to let your students know that in a later video, the teacher asks Sasha and Keoni if x and b are the same or different. They say they are the same.
    - Some class members will notice that Sasha and Keoni focus on the positive value of b and don't include the negative in their equation

8. Homework 3

Thus far, the videos of Sasha and Keoni have been viewed together as a class. For Homework 3, your class members will be asked to watch several video clips from Project MathTalk on their own. It's important that they do so before the next class period, since this work will be foundational for Lesson 4.

Because some of the Project MathTalk videos are being skipped (in the interest of time), Slide 28 situates which videos your class has already viewed, what happened in the ones they are skipping, and what they will watch for Homework 3.