Lesson 1, Parabola Unit
Instructor Notes

Preparation: Create a toolbox for the class. It should contain enough rulers for each small group to have one, several compasses, a few protractors, string, tape, and colored pens or pencils. It's also helpful to have materials in the toolbox that are likely not to be used, so that a solution method is not suggested by the presence of particular tools. These materials could include metal washers (for weight on a string to create a plumb line), plastic triangles with rulers, wire, paper clips, etc.

Lesson:

1. Introduction to the Parabola Unit and to the Construct-a-Parabola Task

Follow Slides 3-11 of the Class PowerPoint. Note the following:

- **Slide 6:** Distribute copies of the Activity sheet to each class member. Ask one or two class members to read the Construct-a-Parabola Task aloud. Save the work of comprehending the task statement for Slides 6-11

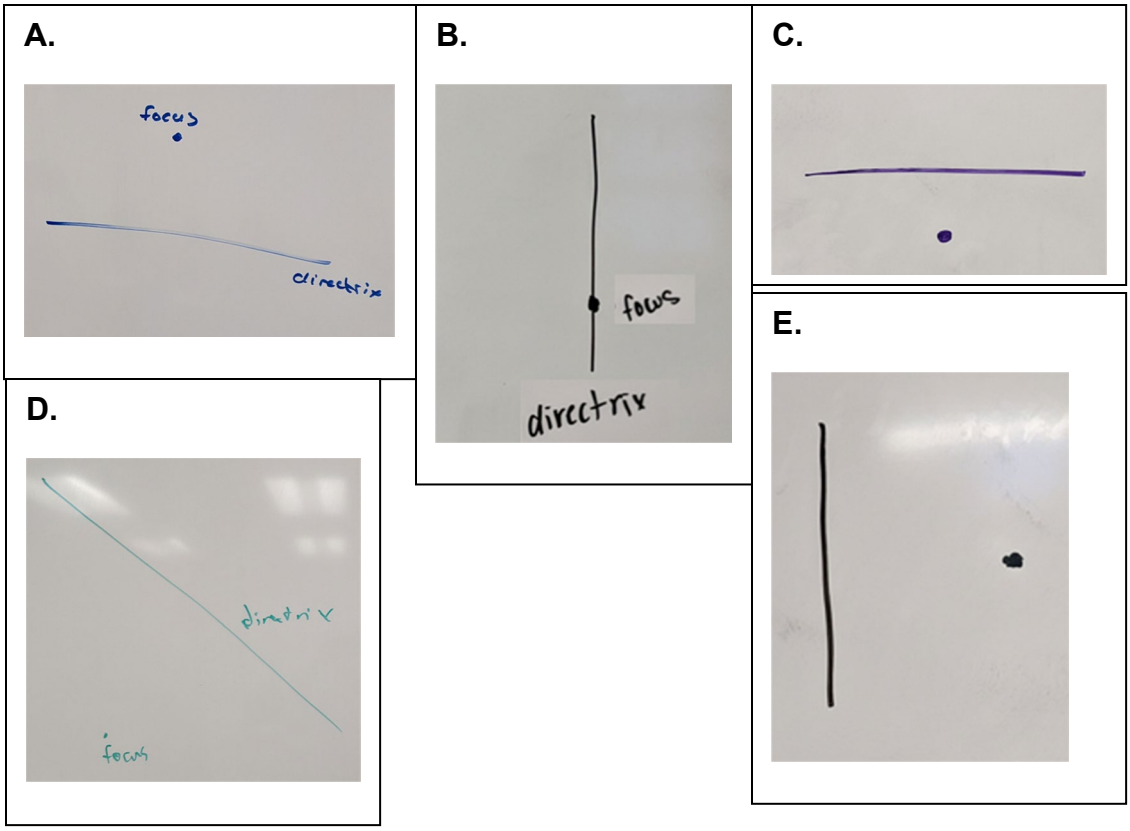
- **Slide 7:** Sample responses:

    - We need a line that is called the directrix and a point that is the focus
    - It doesn't say where the fixed point and fixed line are in relation to each other
    - I'm wondering if the directrix is the line of symmetry of a parabola
    - Is the focus the vertex of the parabola?
    - Each point on the parabola has to be the same distance to the focus as it is to the directrix
    - It's not the case that all points on the parabola are the same distance to the focus as each other.

- **Slide 8**

    Ask a student to go to the board and draw a directrix and a focus. Then ask other students for ALL the possible ways to place a directrix and focus.

    Sample drawings:



Note that A, C, D and E all work. B will not work, which you explore with the class next.

- **Slides 9 and 10**. The goal of these two slides is to explore and eventually conclude that the focus can not be placed on the directrix, even though the definition of a parabola does not explicitly stipulate this.

  Allow class members to explore by trying to place different points on the parabola, when the focus is on the directrix. Then see if the point satisfies the definition, namely that it is the same distance to the focus as that point is to the directrix. After exploring a number of points, students should be able to conclude that the only points that will satisfy the definition lie in a line and will never form a parabola.

- **Slide 10.** The purpose of this whole class activity is to reinforce an understanding of the definition of a parabola before groups start work on the Construct-a-Parabola task.

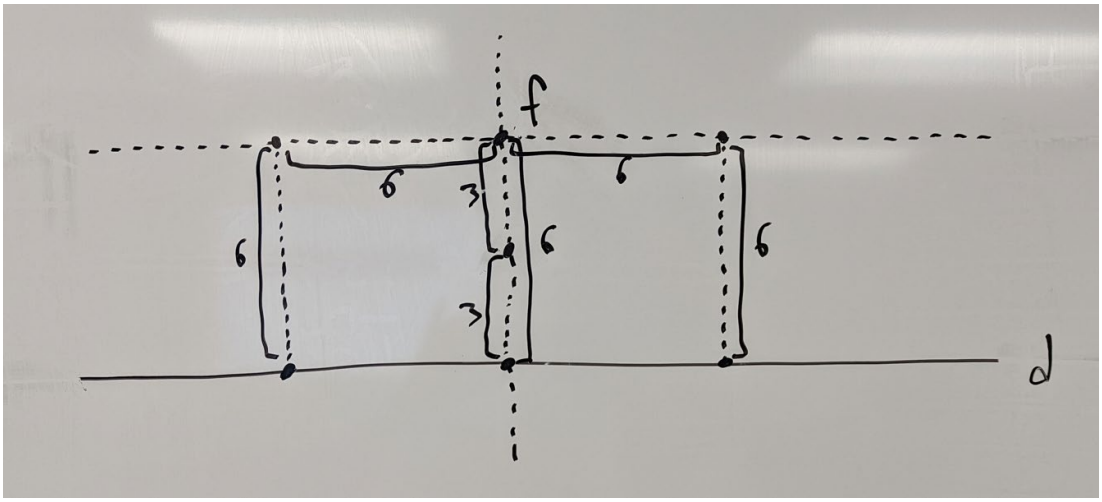
  To respond to the question posed on Slide 10, your pre-service teachers should be asking themselves, “Is the distance from the red point to the focus the same as the distance from the red point to the directrix?” Someone may say that it does fit the definition, because you can draw any distance from the red point to

the directrix. But if we allow that, then any point could be on the parabola. The definition of a parabola assumes that the distance from a point to the directrix must be the shortest distance. In that case, the red point does not fit the definition and is not on the parabola.

2. Groups Work on the Construct-a-Parabola Task

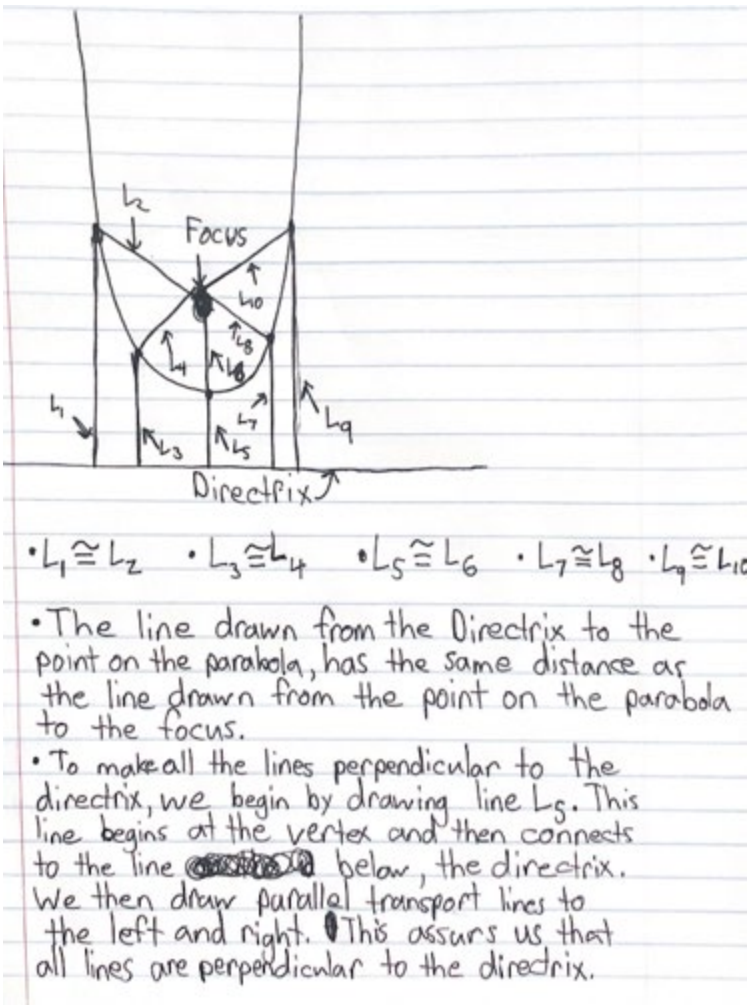
- Put the toolbox on a table in the classroom. Let class members know that they can use any tools they find in it.

- **What to expect.** Class members are likely to find the midpoint between the focus and the directrix. This is often the easiest point to place on the parabola; it will later become the vertex of the parabola. The next easiest points are often those that are aligned horizontally with the focus. The image below shows all three points. Once any of these points are found, you may want to stop the groups working and have class members share. Then resume groups working.



- **Challenges**

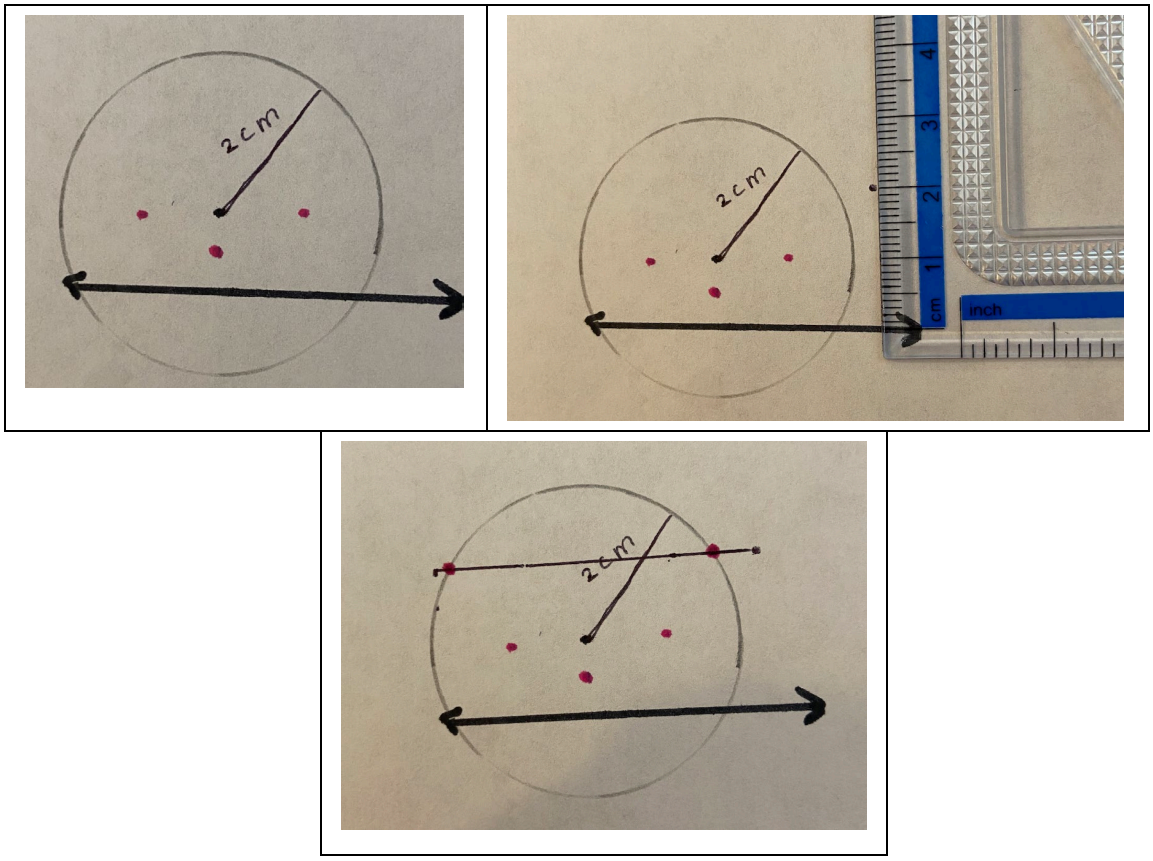
  - Finding a general point on the parabola that satisfies the definition is challenging, even for math majors.
  - Here's work from a student who just placed points in a way that looked like a parabola to them and then declared that for each point, the distance from the point to the focus is the same as the distance from that point to the directrix. It's helpful to ask such a student to get out a ruler and measure the distances to make sure. Then let them know that the challenge of this task is *to create a method to place points* that ensures those distances are the same.



- Some students will try to use algebra or a coordinate grid. It's important to remind them of the instructions and to emphasize that this is a geometry activity; they will connect their approach to algebra later in the unit.

- **Methods**

  - There are many good ways to solve this task. One way is illustrated below. The first image shows a focus that is 1 cm from the directrix. The first three points (shown in red) were constructed using the methods described above. Then to locate a new point, one that is 2 cm from the focus and 2 cm from the directrix, first use a compass to draw a circle with a radius of 2 cm, centered at the focus. That provides the collection of all points 2 cm from the focus. Now to find the 2 points on that circle that are also 2 cm above the directrix, draw a line parallel to the directrix that is 2 cm above it. That provides all the points that are 2 cm above the directrix. Then the two points that intersect the circle and line will be on the parabola. This process can then be repeated for different distances to create additional points on the parabola.



3. Groups share their ideas with the class

4. Homework 1

Depending upon how far groups get in their solution methods, they can continue their work on the Construct-a-Parabola using the Homework 1 instructions. Whatever they generate can be shared with their groups at the beginning of Lesson 2.