# A Bakhtinian Perspective on Learning with Dialogic Mathematics Videos

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The explosive growth in the number of online mathematics videos and the dramatic need for such videos during the COVID-19 pandemic has allowed educators to reimagine how students can learn mathematics. However, the effort to increase access to high-quality learning experiences through online videos has been limited by their uniformity in expository presentation, emphasis on procedural skills, limited attention to mathematical argumentation, and missed opportunities to address common student difficulties (Bowers, Passentino, & Connors, 2012).

In response, our research team created online math videos featuring the dialogue of secondary school students (which can be found at www.mathtalk.org). Alrø and Skovsmose (2004) define *dialogue* as a conversation that involves the quality of inquiry, referring to an interaction that aims to generate new meanings or ways of comprehending. Our videos are unscripted to capture authentic student confusion and resolution of dilemmas. Each video shows a pair of students (called the *talent*) next to their mathematical inscriptions (Figure 1), which allows other students viewing the videos (called *vicarious learners, or VLs*) to see both the talent and their work. "Vicarious" refers to indirect participation in the dialogue of others (Chi, Roy, & Hausmann, 2008). A teacher guides the talent and can be heard but is not seen, so that the focus remains on the talent's reasoning.

Dialogic videos have been used in a small body of research, much of which has focused on quantitative studies of the effectiveness of learning vicariously (e.g., Muldner, Lam, & Chi, 2014). Much less work has sought to understand how this learning occurs. Observing the voicing of common misconceptions seems to play an important role (Muller, Sharma, & Reimann, 2008), as does the inclusion of an authentic learner who displays confusion and asks questions (Chi, Kang, & Yaghmourian, 2017). The purpose of our study is to contribute to this work by investigating the dialogic processes involved as VLs develop mathematical meaning.

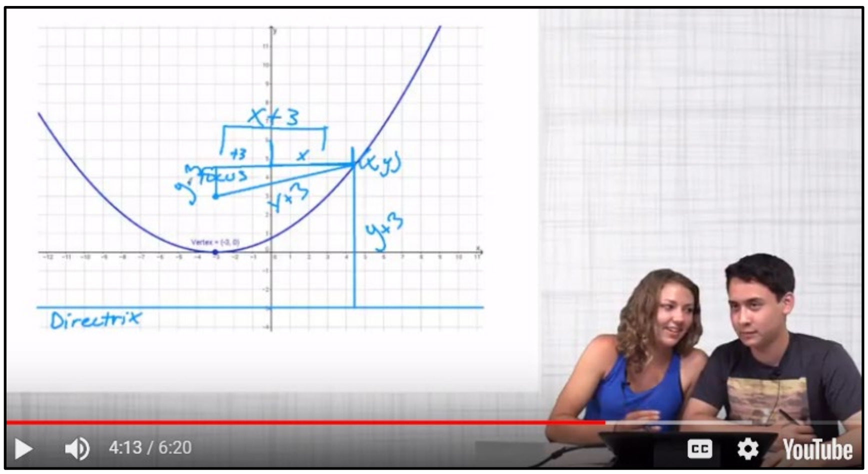



*Figure 1.* Screenshot from an online dialogic mathematics video (from [www.mathtalk.org](http://www.mathtalk.org))

## Theoretical Framework

To investigate dialogic processes, we turned to Bakhtin's (1981) theory of dialogism. According to Bakhtin, everything we say, write, and think is a tapestry of *voices.* Voice refers to an idea, belief, viewpoint, or way of thinking (Kolikant & Pollack, 2015). Voice is not the same as utterance; a single utterance can be multi-vocal. According to Bakhtin, understanding only emerges in the relationship of two or more voices (Kazak, Wegerif, & Fujita, 2015). Specifically, meaning development can occur through *ventriloquation*, which is the process by which one uses words from others but adapts and transforms them to fit into one's own personal narrative. For Bakhtin (1981), words are partly one's own and partly from other speakers: "The word in language is half someone else's. It becomes, 'one's own' only when the speaker populates it with his own intention, his own accent, when he appropriates the word, adapting it to his own semantic and expressive intention" (p. 293).

The process of ventriloquation is not always straightforward. Initially a person may try on words from others by simply repeating them but feel like they are speaking a foreign language (Amhag & Jackobson, 2009). Additionally, there may be resistance or struggle (Taylor, 2003). For Bakhtin, struggle is an essential part of meaning construction, rather than something to be avoided. Finally the negotiation or adaptation of words and ideas from others needs to be integrated into one's personal narrative (Radford, 2000).

## Methods

Two Grade 9 students participated in this study as VLs. Their grades in Algebra 1 were in the B-to-D range. Both were fluent in Spanish and English. The VLs were paired, because research has found that students learn more from viewing dialogic videos collaboratively in pairs than individually (Chi et al., 2008).

The pair participated in 9 research sessions, each lasting 75-90 minutes. The VLs had access to the dialogic videos on a laptop before, during or after working on math tasks. The research sessions occurred in a classroom at the VLs' school after school hours. The researcher sat across the room while the VLs worked. The VLs would call the researcher over to explain their reasoning when they were done or stuck. This meant that the researcher left many areas of confusion unresolved. If the math task in a video was complex, then the VLs worked on the same task. Other times, paired tasks were used (e.g., numerical values were changed for some component of the task), to ensure a high level of problem solving for the VLs.

The overarching goal of the video lessons was to support the derivation with understanding of the vertex form of a general parabola as $y = \frac{(x-h)^2}{4p} + k$, where $(h, k)$ is the vertex and $p$ is the distance from the vertex to the focus. The video unit emphasized connections between the geometric definition of a parabola and algebraic representations. A major theme was *quantitative reasoning*, where a *quantity* is one's conception of a measurable attribute of an object (Thompson, 2011). In the context of parabolas, the quantity of interest is distance. For example, when students construct a parabola from its geometric definition, they have to figure out how to place points so that they are the same distance from the directrix as they are to the focus.

Preliminary analysis started with the creation of descriptive accounts for all 13 hours of videotaped data from the VLs (Miles & Huberman, 1994). To reduce the data to a tractable amount, we identified 9 candidate topics in which the VLs demonstrated an evolution in their understanding. We selected the topic of meaning for the parameter $p$, because of its importance mathematically and its complexity for learners. From the introduction of $p$ to the VLs' development of the meaning of $p$ as a distance was 2.5 hours (Sessions 5 and 6). We identified key episodes in which some meaning for $p$ could be inferred from the VLs work or discourse. Analysis proceeded by applying a Bakhtinian framework to interpret these episodes. In particular, we used the constructs of voice and ventriloquation (including the different aspects of repetition, resistance, and adaptation within one's personal narrative).

**Results**

We focus on two claims (due to space limitations), namely that the VLs engaged in two processes of ventriloquation (during Session 6) – the first involving an alternate conception from the talent, and the second the meaning of *p* as a distance. But first, we set the stage by briefly describing what happened in Session 5.

The general equation of a parabola with vertex at the origin ($y = \frac{x^2}{4p}$) emerged in Session 5 from generalizing a pattern from a set of parabolas that shared a vertex but had different foci (Figure 2). We had intended for *p* to be conceived as the distance between the vertex and the focus. Instead, several voices emerged representing different meanings for *p*, as: (a) the number on the *y*-axis next to the focus in the graph of the parabola; (b) the number multiplied by 4 in the denominator of the equation, and (c) the number added to or subtracted from *y* when deriving the equation.

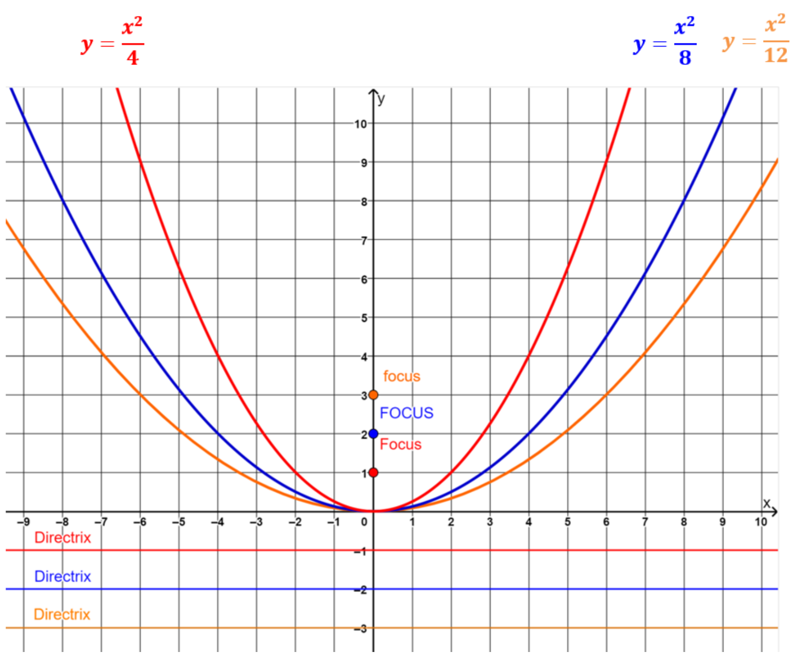



*Figure 2.* Family of parabolas with the same vertex but different foci

**Ventriloquation of "It's a General Place to Put the Focus"**

At the beginning of Session 6, the VLs were given $p = 1.5$ (and the talent $p = ¼$) and asked to find the focus, directrix, equation, and graph for the parabola (Figure 3). However, the VLs first made sense of the talent's task. They watched one of the talent, Keoni, place the focus incorrectly at (0,1) because "it's a general place to put it." The idea of placing by "feel" is disconnected from the meaning of *p* as the distance between the vertex and the focus. Because the distance between the vertex (here, the origin) and the focus is ¼, the focus should be (0,1/4). However, the VLs ventriloquated Keoni's action and reason in three parts:

a. *Repetition of talent's incorrect placement of focus*. The VLs stopped the video and placed the focus at (0,1) on their graph of the talent's parabola (Figure 4).

b. *Adaptation of focus into a personal narrative.* We interpret Bakhtin's notion of a personal narrative in this context to refer to previous successful mathematical strategies and sense-making efforts (from math classes, research sessions, or non-school experiences). The VLs turned to a method they had mastered in previous sessions (i.e., using the Pythagorean theorem combined with the geometric definition of a parabola). They revised the focus to (0,.5), arguing that (0,1) was too close to the point (1,1), apparently because it did not fit the imagery of the right triangle they had in mind. Then they tried to verify (0, .5) using their method.

c. *Repetition of talent's reason.* The VLs restarted the video. When the talent revised their placement of the focus to (0, ¼), the VLs groaned, "We were "way off." One of the VLs explained that she had used .5 because "it felt right to me," providing evidence of ventriloquation, not just of the talent's action, but also of his reason.

| **Talent's Task** | **Vicarious Learner's Task** |
| --- | --- |
| **p = 1/4**<br>• Graph the parabola<br>• What is its equation?<br>• Where is p, the focus and the directrix? | **p = 1.5**<br>• Graph the parabola<br>• What is its equation?<br>• Where is p, the focus and the directrix? |

*Figure 3*. Paired math tasks

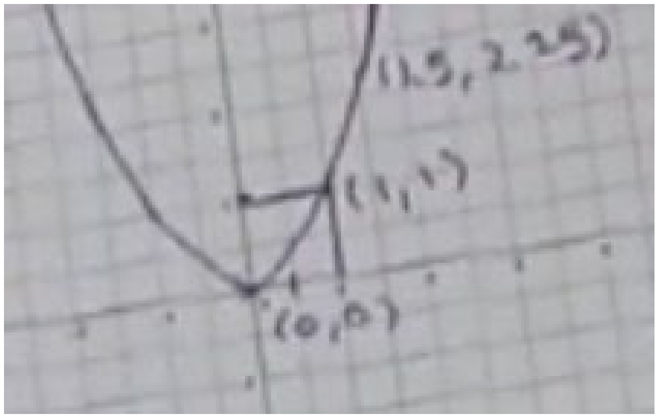



*Figure 4*. Video screen capture of the VLs' placement of the focus at (0,1)

**Ventriloquation of "*p* as a Distance"**

The VLs viewed a video twice in which the talent resolved the dilemma of placing the focus, connected the *p*-value to the focus, and talked about *p* as the distance from the origin to focus and from the origin to the directrix. The VLs responded as follows:

a. *Setting aside p as a distance (resistance).* Even after viewing the video twice, the VLs said they were confused. They temporarily set aside the voices from the video.
b. *Adaptation of talent's connection into personal narrative.* The VLs turned to something more familiar, graphing the parabola in their paired task (where *p* was given as 1.5) by creating a table of values. At the end of a lengthy process, they reread the task statement, which asked them to label the focus, directrix and *p*. One VL said that Keoni must have been right that ¼ was the focus of the first task; the other VL placed one finger at 1.5 on the y-axis and another finger at -1.5, and tapped the inscription "*p* = 1.5" at the top of the task sheet. Their subsequent labeling of the focus and the directrix on the graph suggests an adaptation of the talent's connection between the p-value, focus and directrix into their personal narrative of a tabular approach to creating the parabola. However, they did not label *p* or speak of it as a distance.

c. *Emergence of the meaning p as a distance.* The researcher asked where *p* is on the graph. After a long pause, one VL gestured in a sweeping motion (indicating extant or distance) from the vertex to the directrix and then from the vertex to the focus. She labeled each as *p* (Figure 5). When asked to solve a similar task for *p* = ½, the VLs reversed their approach, this time starting with the *p*-value, labeling the focus and directrix, and finishing with the table of values (Figure 6). One VL, Brenda, stated: "Since we know ***p*** **is the distance between the focus and the directrix**, we first put point 5…then we solved for the points." Thus, the VLs appear to have integrated the voice of *p* as a distance, from the video into their personal narrative.

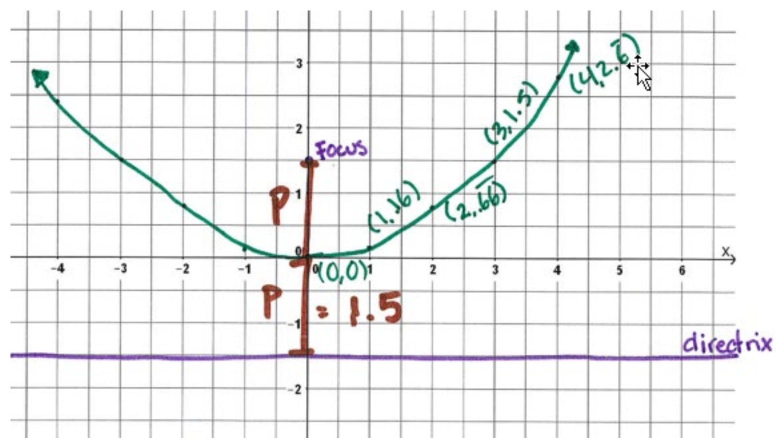
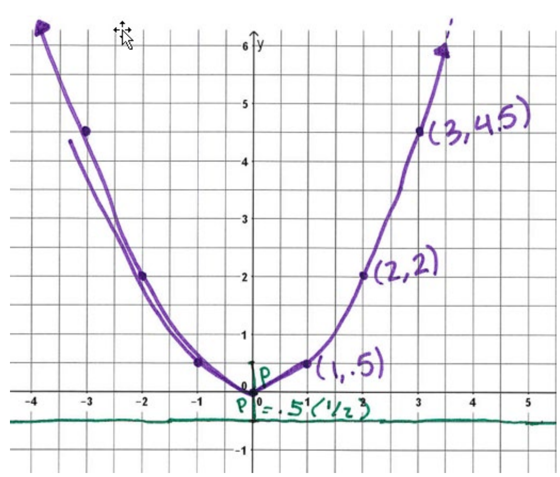



*Figure 5*. The VL's placement of *p* on a graph



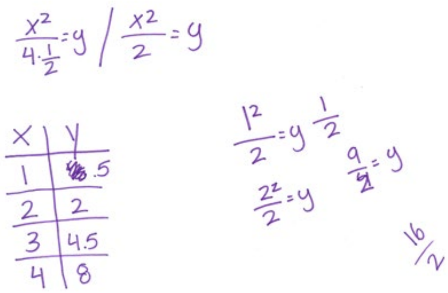



*Figure 6*. The VLs solution to the task in which $p$ = ½

## Discussion

This Bakhtinian analysis extends previous research on the importance of misconceptions and authentic learners in dialogic videos by providing insights into *how* VLs make use of the talent's alternative conceptions and of their resolutions through the process of ventriloquation. Our study also expands the investigation of ventriloquation in mathematics education research. Previously, ventriloquation had been applied to curricular reform messages (Graue & Smith, 1996), cultural narratives (Svensson, Meaney, & Noren, 2014), and voices surrounding mathematical identity (Solomon, 2012). Instead, we explored ventriloquation as a learning process. Specifically, we built upon earlier instances of the ventriloquation of individual words at a single phase (i.e.., Taylor, 2003; Radford, 2000) to present a progression through phases of ventriloquation from repetition to resistance to the adaptation of voices into a personal narrative.

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