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Project MathTalk

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Session Abstract
In response to the prevalence of talking heads or hands in online math videos for K-12 students, the research program featured in this symposium (Project MathTalk; www.mathtalk.org) created videos that feature student dialogue. We present the results of three studies of secondary school students and preservice secondary teachers learning from these videos. Issues that emerge from the use of alternative online videos will be discussed with the audience.

Introduction to Project MathTalk

Despite the enormous number of mathematics videos available online, there is surprising uniformity in the use of talking heads or hands to demonstrate step-by-step procedures using a traditional pedagogical approach (Bowers, Passentino, & Connors, 2012). A review of online mathematics videos revealed that few videos created for K-12 student learning (versus for professional development with teachers) included dialogue with children (Lobato, Walters, & Walker, 2016). Either children mimicked the script of a traditional teacher, or in the few videos in which problem solving occurred, animated characters were used rather than real children. Alternative videos are needed that capture student dialogue as characterized by Alrø and Skovsmose (2004) as a conversation that involves the *quality of inquiry*, meaning that there is an interaction that aims to generate new meaning or to open up different ways of experiencing things.

In response, the NSF-funded research program featured in this research symposium, Project MathTalk, created and is now investigating the use of unscripted videos in which secondary school students convey sources of confusion and resolve their own dilemmas by arguing for and against particular ways of reasoning (see Figure 1 for a screenshot from one of the MathTalk videos; www.mathtalk.org). Our program of research is framed by the theoretical assumption that dialogue is central to learners' enculturation into forms of academic argumentation and that it mediates thinking (Vygotsky, 1978). Although dialogue is well accepted as an important tool for learning, the efficacy of *watching* dialogues has been a matter for debate. There is some evidence, from the domains of physics, computer literacy and communications, that students who vicariously observe a dialogue outperform those who observe a monologue (Craig, Sullins, Witherspoon, & Gholson, 2010; Fox Tree, 1999; Muller, Bewes, Sharma, & Reimann, 2008). On the other hand, in one of the few such studies in mathematics, the vicarious learners did not use the video spontaneously when asked to solve a related task (Kolikant & Broza, 2011).

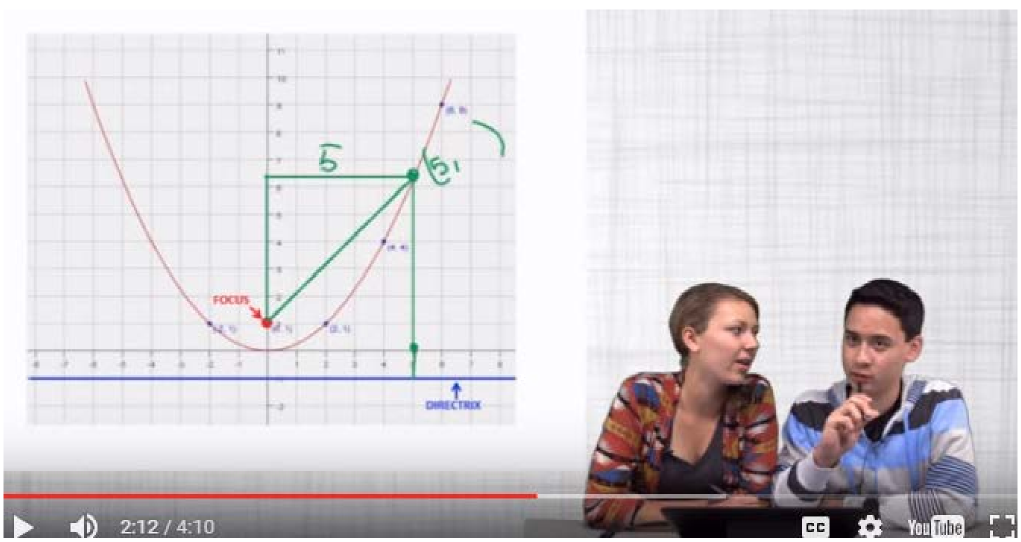

Figure 1. Screenshot of a MathTalk Video

This research symposium presents findings from three studies conducted using the dialogue-intensive videos created for Project MathTalk. The first two papers convey results from the use of the videos by high school students—13 pairs of students using 1 video-based lesson

(Paper 1) and 1 pair of students using 8 video-based lessons (Paper 2). Paper 3 examines the mathematical knowledge for teaching (MKT) that preservice secondary teachers (PSTs) can develop from the video-based lessons.

There are three potential benefits this symposium offers the field of mathematics education. First, by presenting research evidence for the promise of an alternative model of online math videos, we hope to inspire developers of video-based tools to re-imagine the possibilities of online learning. Second, by discussing emerging issues associated with non-traditional videos, we hope to arm teachers with strategies for engaging their students in dialogue around videos that demonstrate the struggle and persistence that is part of authentic mathematical practice. Finally, we hope the symposium serves as a launching point for creating research questions for future investigations into learning by observing the dialogue of others.

Paper 1: The Games Vicarious Learners Play

Carren Walker, Matt Voigt, Joanne Lobato & C. David Walters
San Diego State University

There is an emerging literature on *vicarious learning*, which refers to learning by observing and engaging with video- or audio-taped presentations of other people engaged in learning (Chi, Roy, & Hausmann, 2008). Most of this research has focused on quantitative studies of the effectiveness of learning from watching dialogues and has been conducted at the undergraduate level (e.g., Driscoll et al., 2003; Gholson & Craig, 2006; Muller, Sharma, Eklund & Reimann, 2007). In contrast, this paper examines the qualitatively different ways in which secondary school students approached vicarious participation in a video dialogue among their peers.

Data were collected from 13 pairs of students from a diverse high school (87% free and reduced lunch, 46.2% English learners) in a southwestern U.S. city. Each pair participated in a video-taped 90-minute mathematics session in which they viewed a 25-minute video-based lesson (split into 3 episodes). In the videos, the talent worked together to make sense of the geometric definition of a parabola (i.e., a parabola is the set of points that are equal distance from a fix point, called the focus, and a fixed line, called the directrix) and to create a parabola from the definition. The vicarious learners (VLs) were asked what they noticed in each video episode and also to create a parabola using the definition.

Our analysis is inspired by table or board games, which involve a form of play or activity that has a goal and is engaged in according to a set of rules. Whereas the goal of a game is the overarching aim or target (what one is trying to accomplish), a rule is a statement of a boundary or regulation that both facilitates and constrains player behavior in a game. The set of rules governing a game describe the relationship between the players and the other elements of the game (Wood, 2012).

Unlike board games in which the goal and rules are provided before the activity begins, the games that the vicarious learners (VLs) appeared to play in our study seemed to emerge**,** often while VLs were viewing and reflecting on the first two video episodes (and before they began the main math task of creating a parabola from its geometric definition). The goals and rules are also products of our inferences, rather than being given to players at the outset.

The result of our analysis is that the VLs appeared to play four different games: (a) the definition game; (b) the concept image game; (c) the procedure game; and (d) the expert game. Each game can be characterized by a goal and three rules that pertain to three different key elements of the activity – the math videos, the tasks that VLs were asked to work on, and the researcher who interacted with the pair of VLs during the session (as shown in Figure 2). Specifically, each game has a rule for how VLs obtained information from the videos, a rule for how VLs created a response to the task, and a rule for how the VLs justified that response to the researcher. The arrows are uni-directional because they indicate that the rules describe the behavior of the player. Figure 2 only captures the structure of the ruleset and goal; it doesn't capture everything that happens during the game. For example, by using a uni-directional arrow from the player to the researcher, we are not saying that the researcher never influences the player. Rather the justification rule describes the actions the player takes in justifying his/her solution method to the researcher.

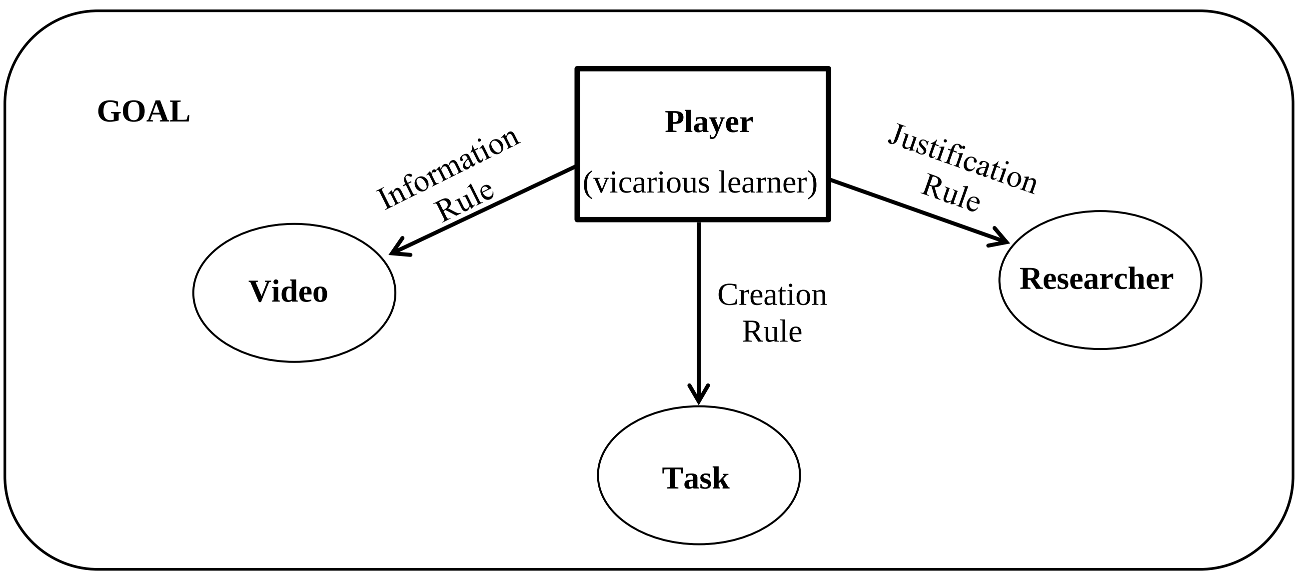

Figure 2. Each game played by the vicarious learners had an emergent goal and rules that pertain to different key elements of the activity

For three of the games (the definition game, concept image game, and procedure game), the element of the task is to create a parabola given its definition. That is, these three games pertain to mathematical activity. However, for the expert game, the main task for the VLs seemed to be the design of the videos. Thus, the goal of the VLs playing the expert game was to provide advice and feedback about the design of the online math videos. Additionally the rules that seemed to govern their behavior pertain to analyzing the design features of the videos (e.g., use of color, font size, sound, amount of information, etc.) and recommending how the researchers can fix the videos. When the researcher insisted that the particular pair playing the expert game actually construct a parabola, they appeared to shift to another game (namely the concept image game), presumably since the expert game pertains to video design rather than to mathematical activity. The goals and rules for each game follow in Table 1. The game framework provides an analytic tool through which to understand the variety of ways in which the VLs oriented themselves to the mathematical session through a non-deficit lens.

Table 1.

The Four Games Played by the Vicarious Learners, with the Emergent Goal and Rules

GAME	Goal	Information Rule	Creation Rule	Justification Rule
Definition Game	Produce points that fit the definition of a parabola.	Make sense of information from the video that either shows methods or allows you to create your own method for placing a point so that it is the same distance to the focus as it is to the directrix.	Generate points using any method (one from the video or one of your own) so that eah point satisfies the definition of a parabola. You do not need to use the same distances (e.g., between focus and directrix) as the talent.	Justify a potential point being on the parabola if it is the same distance from the focus as it is from the directrix; reject the point if it is not.
Concept Image Game	Produce a drawing that matches your concept image of a parabola, which in most cases is a U-	Adapt information from the video so that it conforms to your concept image; reject or ignore information that doesn't	Create points using any method that produces a parabola that fits your concept image. You may draw the parabola freehand, make a coordinate	Justify a potential point being on the parabola by checking to see if it fits your concept image; reject a point if it

	shaped curve on a coordinate grid that opens up or down.	conform.	grid from scratch to help you place points so that the parabola looks right, or some other method.	doesn't fit.
Procedure Game	Participate in the activity until a procedure (from the video) emerges; then follow that procedure to produce a correct solution (which may be to reproduce the talent's parabola exactly, rather than to make any parabola that fits some condition).	Extract information from the video that shows the steps that the talent are following. Stop the video a lot and take notes or make drawings.	Do exactly what the talent do to make a parabola. You may want to use the same measurements as the talent.	Ask the researcher if you are doing it correctly or compare your drawing to the talent's to see if it's the same.
Expert Game	Provide advice and feedback about the design of the math videos.	Analyze elements of the videos: ▪ Background noise ▪ Color of animations ▪ How things look (too many dots, not enough lines, etc.)	Make suggestions for how to improve the videos. If you don't understand something in the video, you can figure out a way that the videos could be created to make them more understandable.	Justify your recommendations to the researcher by inferring how other people will receive the videos.

Paper 2: A Learning Trajectory for Vicarious Learners

Joanne Lobato, San Diego State University

The study presented in Paper 1 used videos lasting less than an hour, as have most investigations of vicarious learning. Even though significant findings have emerged from short treatments, such studies are limited in the claims they can make regarding the nature of vicarious learning over time. Thus, the study presented in this paper investigated the following research question: What is the learning trajectory of a pair of vicarious learners using an entire unit of video-based lessons?

Data collection proceeded in a similar manner as for the study presented in Paper 1 but was repeated for nine 90-minute sessions, covering 8 video-based lessons from Project MathTalk's unit on parabolas. The unit begins with the talent creating a parabola from its geometric definition. Then over the course of many lessons, it builds the machinery so students can derive the vertex form of a general parabola, which is $y = \frac{(x-h)^2}{4p} + k$ where the (h,k) is the vertex and p the distance from the vertex to the focus. A major theme of the unit is *quantitative reasoning*, where we follow Thompson (1994) in characterizing a quantity as one's conception of a measurable attribute of an object. In the context of parabolas, the quantity of interest is distance. When students grapple with how to create a parabola from its geometric definition, they have to figure out how to place points so that they are the same distance from the directrix as they are to the focus. Then as they are creating equations for parabolas on a coordinate grid, they need to represent distances using algebraic expressions such as y + 1. Thus, quantitative reasoning in the parabola unit encompasses connections between geometry and algebra.

One pair of vicarious learners, Desiree and Belinda, was recruited from the study described in Paper 1. They were selected because they played the definition game in the first study. In their regular Algebra 1 class, they were earning in the B to D range. Both were fluent in Spanish and English.

The same researcher interacted with both the talent and the VLs, but her role changed. When we designed the VL study, we considered two ends of a spectrum for the role of the researcher. On the one hand, we could try to recreate circumstances of an individual accessing these videos online at home, with little or no interaction with a researcher. However, we worried we wouldn't get a rich verbal trace to examine. On the other hand, the researcher could play the same role as she did when teaching the talent. However, we wanted the videos to serve as the primary source of instruction. Thus, we limited the researcher's actions to giving praise, presenting tasks to students, and framing the activity so that VLs could watch the videos before, during or after working on the task. Sometimes the tasks were identical to those in the video, but to ensure a high level of problem solving for the VLs, we sometimes used paired tasks where the task given to the VLs differed slightly from the one the talent worked on, e.g., the parabola had different vertex or the axes were scaled differently. The researcher sat across the room while the VLs worked on the math tasks and watched videos. The VLs would call the researcher over to explain their reasoning to her when they were stuck or done. This meant that the researcher left many areas of confusion unresolved; in this respect the sessions resembled an interview.

The first step in analysis was to identify major macro-level components of the VLs quantitative reasoning trajectory. First they spent several sessions (i.e., Sessions 1-5) working out how to measure lengths through cycles of first ignoring and then attending to the scale of the axes. Second, during Session 5 and 6, they made sense of the parameter p as a quantity of length. Third, they learned to relate algebra expressions, such as $y - 3$, to distances in Sessions 4-6 and 9. For each of these components, retrospective analysis of the video-taped sessions was performed to characterize the VLs' conceptions and modifications of their conceptions over time based on our inferences of regularities in students' actions, by following Clements and Sarama (2009) and by drawing upon open coding from grounded theory (Strauss & Corbin, 1990).

The development of the VLs' quantitative reasoning is a complex result. We focus on a part of the third component (relating algebraic expressions to distances) here. Specifically we will provide evidence to support the following claims:

1. Initially the VLs talked about the meaning of y (in an expression like y + 3) as any distance in a vertical direction. As a result, the VLs attributed several different algebraic terms to the same length.
2. Eventually they appeared to conceive of y as the distance from a point (x,y) to the x-axis, which allowed them to differentiate lengths represented by different algebraic terms.
3. As part of this transition, there is a pivotal moment in which the VLs went to the videos to help them make progress.

The VLs were first asked about their meaning for y in Session 4. The VLs had drawn a line segment down from the general point (x, y) to the directrix (y = -2) and expressed the length of the segment as y + 2. The researcher asked, "In this [pointing to the y + 2], there's a y, a 2 and a y plus 2; Where's the y? The distance y?" In response, one of the VLs gestured up and down the y-axis. In the next similar task where the distance between (x, y) and the directrix was y + 3, the VLs gestured up and down a vertical line to the right of the y-axis, and said, "This is all y." Thus, y appeared to shift slightly from representing the y-axis to being any vertical distance. As a result, when the VLs were explaining the meaning of the different algebraic expressions that they had labeled on a graph (shown in Figure 3), they created line segments with distances that we recognize as y, y-p and the y-axis, but they attributed each length to y. When the researcher asked for clarification, the VLs stated explicitly that "y is everything" and that "y is vertical," as can be seen in the transcript excerpt below:

Researcher: The one thing I just want to clarification on is that I've heard that y is this, this and this [points to each of the annotated distances from Figure 3, in turn]. Is that true?

Desiree: y is everything! (laughs)

Belinda: (laughs)

Researcher: y is everything.

Belinda: y is just everything that's [gestures up and down on graph with hand], uh

Desiree: You see y over here [places pen near the y in y + p in the middle of the graph]. You see y over there [places pen on the far right vertical line connecting (x,y) with the directrix]. You see it down there [places pen on y-axis below the origin]. You see it everywhere.

Belinda: [Moves hand up and down over graph] vertical; it's everything that's vertical [continues to move hand up and down].

Researcher: But for p, where is it?

Desiree: p is anything that is above [places pen at origin], it's like from the focus and the origin [places finger at the focus and pen at the origin, and then moves finger up and down between the two locations], that's p. That's something you're subtracting.

Researcher: Ok. All right. That's very helpful.

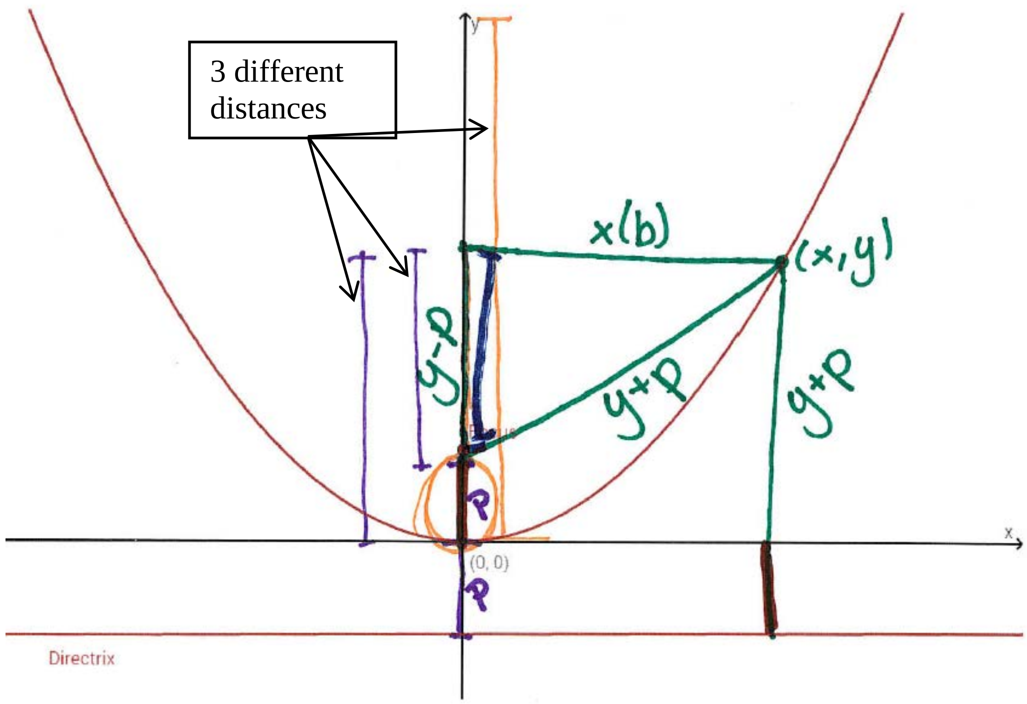

Figure 3. Author's annotation of 3 different distances marked on the graph by the VLs, and all attributed to y.

The verbal statements, along with the numerous vertical gestures suggest, that the VLs conceived of the quantitative meaning for y as being any vertical distance. It's interesting to note that they interpreted p correctly, which suggests that their difficulty is not with the interpretation of literal symbols in general. Their meaning for y as any vertical distance persisted into Session 9, the final session of the study.

Then there appeared to be a breakthrough. The VLs were working for the first time with a parabola that had been translated to the right from the origin (see Figure 4). The VLs represented the length of the horizontal leg of the right triangle incorrectly as b, rather than as $x - 10$. [Note that both the talent and the VLs used b and x interchangeably; the b comes from the way the Pythagorean Theorem is often expressed.] They created an equation but determined that it was incorrect by substituting values. This was a fairly lengthy episode that transpired while the researcher was sitting across the room. Desiree explained the nature of the problem to Belinda, by noting that their equation "is for the origin, but our origin [meaning vertex] is (10,0). So that's what changes everything. So this cannot work." She then spontaneously started the video.

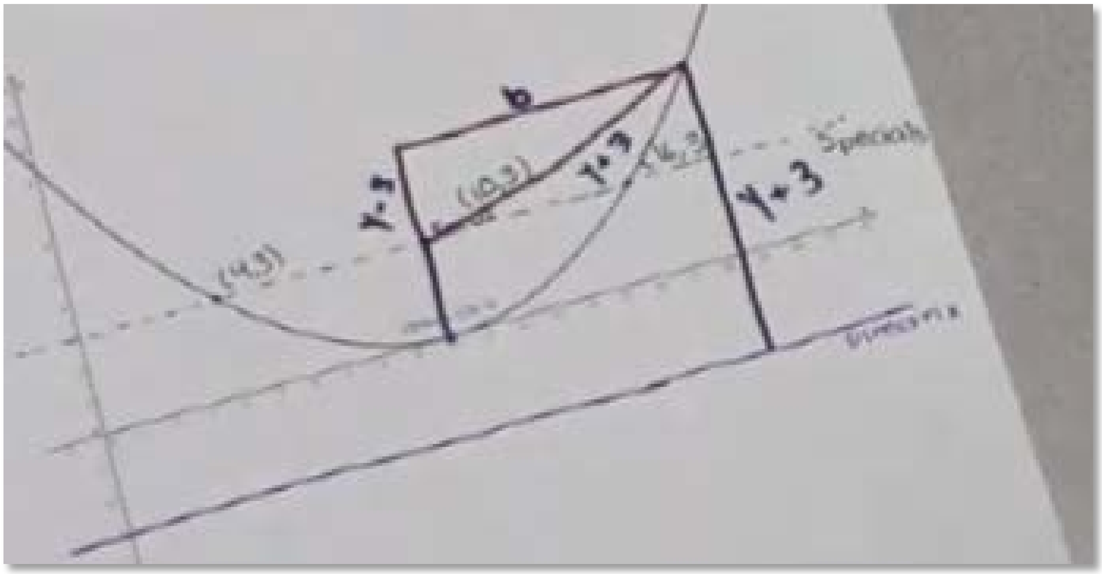

Figure 4. The VLs incorrectly expressed the length of the horizontal leg of a right triangle as *b*, on a parabola with vertex (10, 0).

In the video episode that the VLs watched, the talent were trying to determine the length of the horizontal leg of a right triangle for a paired task, in which the vertex is (7,0) rather than (10, 0). At first the VLs silently watched as the talent (Sasha and Keoni) wrote the correct length as x -7, but as soon as Sasha explained her reasoning for subtracting 7 units from the distance between the point (x, y) and the y-axis, Desiree groaned loudly and dropped her head into her hands on the table. She turned to Belinda and said, "Doesn't that speak to you?" which suggests some readiness for taking up the talent's reasoning. Desiree went on to bemoan, "Why did we not think of that?" She then announced, "We found the mistake!" The VLs corrected the length of the horizontal leg of their right triangle to x – 10 on their paired task. When the researcher asked them, "What did you notice in the video that helped you?", the VLs responded in unison, "the subtracting 7." Belinda went on to reason in a manner similar to Sasha by saying, "Because now that it's not on the axis, we have a distance right there [places two fingers so that they span the distance between the y-axis and the leftmost part of the right triangle] and we have to subtract that distance. She then correctly identified the lengths that are represented by x and x – 10.

This episode seemed to have an effect on how the VLs conceived of the meaning of y. Later in the same session, the researcher returned to a task similar to the one shown in Figure 3. The VLs were finally able to correctly identify y as the distance from (x, y) to the x-axis and differentiate it from the distances represented by y – 3 and 3.

Our ongoing analysis suggests that the quantitative reasoning trajectory of the VLs followed the general arc of that for the talent, but that the VLs had some unique challenges as well as different strengths. For example, both the talent and the VLs made sense of terms such as y, y – 3, x, and x – 7 in terms of distance. But the meaning for y did not appear to be problematic for the talent, whereas working out the meaning of y took several sessions for the VLs. On the other hand, the data show that there were instances when this type of result was reversed, e.g., the talent struggled to make sense of (x, y) as a representation for any point on the parabola, whereas the VLs did not seem to have any difficulty with this.

Reflecting upon what has been presented here suggests the potential power of dialogue-intensive videos. The VLs appeared to resolve a struggle with the meaning of y by watching a video of the talent in dialogue, even though the dialogue was about x – 7 not y. Furthermore the researcher didn't provide information directly about the meaning of y; in fact, she found it challenging to let the confusion go for five sessions. The VLs were able to pick up a way of reasoning that was useful to them from a discussion of different quantities by the talent.

Paper 3: Training Prospective Teachers Using Online Videos That Feature Student Dialogue

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Silverman and Thompson (2008) proposed a theoretical framework for the development of mathematical knowledge for teaching (MKT) in which the foundation for MKT is an individual's own rich mathematical understandings. Teuscher, Moore, and Carlson (2016) added to this framework by suggesting that one way to think about MKT development is through the construct of decentering. This Piagetian construct was originally developed to describe a child’s actions as he or she considered thoughts, feelings, or perspectives that were different from his or her own (Piaget, 1955). In simplest terms, decentering is a process through which an individual metaphorically sets aside his or her own understanding of a situation in order to understand the situation from the perspective of another individual (Teuscher et al., 2016).

Teuscher et al. (2016) argued that decentering is a key component in the formation of MKT by linking the construct to the field’s notions of first and second-order models. According to Steffe and Olive (2010), first-order models are “the models an individual constructs to organize, comprehend, and control his or her experience, i.e., their own mathematical knowledge” (p. 16). By contrast, a second-order model is an image of another individual’s ways of understanding a mathematical situation (Teuscher et al., 2016). Second-order models are constructed via decentering when a teacher turns away from her or his own understanding to try to make sense of how their students are making sense of a mathematical situation. Teuscher et al. argued that it is through the building up of second-order models that teachers develop MKT.

One consequence of conceptualizing MKT in this manner, is that it emphasizes engagement and experience with authentic student mathematical activity. In other words, to develop MKT, teachers necessarily must have opportunities to observe, interact with, and reflect on students' mathematical thinking. While in-service teachers may have many such opportunities each day to do just that, prospective secondary teachers (PSTs) often have little experience with taking on the role of a teacher as they enter teacher training programs.

One potential solution to the problem of providing PSTs with opportunities to engage with and reflect on student thinking is the use of online math videos. The use of video in teacher education is well documented (Borko, Jacobs, Eiteljorg, & Pittman, 2008). However, videos used in teacher education tend to be minimally-edited (Borko, Koellner, Jacobs, & Seago, 2011) and tend to provide a flood of information that easily obscures the phenomena meant to be observed by the PSTs who view them (Erickson, 2007). Moreover, many online videos tend to be brief "snapshots" of students' learning at a certain moment in time (e.g., the videos used in Cognitively Guided Instruction training, Carpenter, Fennema, Franke, Levi, & Empson, 1999), rather than showing the development of particular students' thinking over time.

If MKT development requires sustained engagement with student thinking so that teachers develop images of students' thinking and understanding over time, then videos used in PST training should address both the issues just discussed. The MathTalk videos that I have helped develop do just that. First, they are heavily-edited to focus the viewer's attention on two students (referred to as *the talent*) who develop several mathematical relationships between geometric and algebraic conceptions of parabolas. The videos feature animations, annotations, and voice-overs to highlight the students' mathematical development (Lobato, Walters, & Walker, 2016). Second, the MathTalk videos show the same pair of students over the course of several lessons. This longitudinal feature of the MathTalk videos provides viewers with a coherent story of how the talent's thinking developed over time.

The design of the experiment was driven by the following research question: What is the nature of the MKT that develops for PSTs during a video-based mini-course? Seven students from two different large Southwestern universities participated in a 12-hour mini-course. The students were all upperclassmen majoring in mathematics and seeking certification as secondary mathematics teachers. The mini-course featured the use of the MathTalk videos, as well as mathematical tasks that were similar to the ones featured in the videos. Pre- and post- interviews (Ginsburg, 1997) were conducted with each participant to assess the nature of knowledge that developed during the mini-course.

During the post-interview, I had participants solve the Parabola Task, the goal for which was to use the geometric definition of a parabola to generate the general form of an equation for a parabola. After participants solved the task, I asked them a follow-up question about how they thought the talent would have solved the task. Despite having never seen the talent solve the task, all seven of my participants correctly predicted how the talent solved the task (the talent's solution is in Lesson 9 of the MathTalk parabola unit). Four of these participants initially solved the task like the talent.

Three of my participants experienced dramatic shifts in their ability to take on the perspective of high school students in that they initially solved the task in ways that were dissimilar to the talents' method, yet in response to my follow-up, they were able to discuss in rich detail how they thought the talent would approach the task. For example, one participant named Marshall initially provided two solutions for the task, including one in which he talked about horizontal and vertical shifts in the plane to derive an equation, and the other in which he relied on sophisticated algebraic reasoning and the distance formula to derive another equation.

When I asked Marshall how he thought the talent would solve the task, he accurately predicted how they would approach the task. He also provided rich details about their reasoning and how they would struggle and overcome those struggles. Marshall was able to do this despite having never had live interaction with the talent or seeing them solve the Parabola Task. This can be taken as evidence that Marshall, and the other participants who experienced this shift, were able to build second-order models through their engagement with and reflecting on the MathTalk videos. In particular, the videos seemed to serve as a powerful substitute for live experiences with students and their thinking, and the videos acted as a powerful tool for helping participants learn to decenter.

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